THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH4240 - Stochastic Processes - 2022/23 Term 2

Homework 1
Due date: January 20 Friday, 2023

Submit your answers in a single PDF file **online via Blackboard**. The late submission will not be accepted. Reference solutions will be provided after grading.

(1) Let X and Y have the joint probability density function given by

$$f_{X,Y}(x,y) = \begin{cases} 6(1-y) & \text{if } 0 \le x \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the marginal density function $f_X(x)$ for X.
(b) Find the conditional density function of X given $Y = y$.
(c) Are X and Y independent? Explain why or why not.
(d) Find $P(Y \ge \frac{3}{4} | X = \frac{1}{2})$.
(e) Find $E(X - 3Y)$.

(2) Consider two independent random variables X and Y. The pdf of X is given as

$$P(X = i) = \tfrac{1}{3} \text{ for } i = -1, 0, 1,$$

and the pdf of Y is given as

$$f_Y(y) = \begin{cases} 1 & \text{if } 0 \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

Define $Z = X + Y$.
(a) Compute $P(Z \le \frac{1}{2} | X = 0)$.
(b) Find the pdf of Z.