THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH3310 2022-2023
Homework Assignment 1
Due Date: September 26, 2022

1. Solve the following ODE using method of integrating factor

$$x^4y' + 5x^3y = e^{-x}, \quad x < 0$$

with condition $y(-1) = 0$.

2. Solve the following second order ODE using method of integrating factor

$$-2y'' + 4y = 8x^2 + 13x - 11$$

with conditions $y'(0) = 0$ and $y(1) = 4$.

3. Please show that

$$\int_0^{2\pi} \cos kx \cos mx \, dx = \begin{cases} 2\pi, \text{ if } k = m = 0 \\ \pi, \text{ if } k = m \neq 0 \\ 0, \text{ if } k \neq m \end{cases}$$

and that

$$\int_0^{2\pi} \sin kx \sin mx \, dx = \begin{cases} 0, \text{ if } k = m = 0 \\ \pi, \text{ if } k = m \neq 0 \\ 0, \text{ if } k \neq m \end{cases}$$

where m, k are non-negative integer.

4. Let $f(x) = x^2$, then please compute the Fourier series of $f(x)$ on $[-1, 1]$.

5. Find the Fourier series solution to the differential equation

$$y'' + 2y = 3x$$

where $0 \leq x \leq 1$ and $y(0) = y(1) = 0$.

6. Solve the following PDE using Fourier series

$$\begin{cases} u_t(t, x) = 4u_{xx}(t, x), \quad 0 < x < \pi, t > 0 \\ u_x(t, 0) = 0 = u_x(t, \pi), \quad t > 0 \\ u(0, x) = f(x), \quad 0 \leq x \leq \pi \end{cases}$$

where $f(x) = x$.