THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH 2058 Honours Mathematical Analysis I 2022-23
Tutorial 8
3rd November2022

- Tutorial problems will be posted every Wednesday, provided there is a tutorial class on the Thursday same week. You are advised to try out the problems before attending tutorial classes, where the questions will be discussed.
- Solutions to tutorial problems will be posted after tutorial classes.
- If you have any questions, please contact Eddie Lam via echlam@math.cuhk.edu.hk or in person during office hours.

1. For each of the following limit, if it exists, find its value (including $\pm\infty$) and prove using $\epsilon - \delta$ definitions; otherwise explain why limit does not exist.
   (a) $\lim_{x\to 1^+} \frac{x^2}{x-1}$.
   (b) $\lim_{x\to 1} \frac{x^2}{x-1}$.
   (c) $\lim_{x\to 0} \frac{\sqrt{x+1}}{x}$.
   (d) $\lim_{x\to \infty} \frac{\sqrt{x+1}}{x}$.
   (e) $\lim_{x\to \infty} \frac{x-\sqrt{x}}{x+\sqrt{x}}$.
2. Let $A \subset \mathbb{R}$, $f : A \to \mathbb{R}$ and $c \in D(A) \setminus A$, assume that $f > 0$ on A, prove that $\lim_{x\to c} f = \infty$ if and only if $\lim_{x\to c} \frac{1}{f} = 0$.
3. Let f be defined on $(0, \infty)$, prove that $\lim_{x\to\infty} f(x) = L$ if and only if $\lim_{x\to 0^+} f(\frac{1}{x}) = L$.
4. Let f, g be functions defined on $\mathbb{R}$, suppose that $\lim_{x\to\infty} f = L$ and $\lim_{x\to\infty} g = \infty$, prove that $\lim_{x\to\infty} f \circ g = L$.
5. For any $x \in \mathbb{R}$, denote $\lfloor x \rfloor$ the floor function, which outputs the greatest integers smaller than or equal to x (e.g. $\lfloor e \rfloor = 2$, and $\lfloor -1.5 \rfloor = -2$.) Determine the points of continuity for the following functions.
   (a) $\lfloor \sin x \rfloor$.
   (b) $\lfloor 1/x \rfloor$, where $x \neq 0$.
6. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is continuous at c, and $f(c) > 0$. Prove that there exists a neighborhood of c on which f is positive, i.e. $\exists \delta > 0$ such that for x satisfying $|x-c| < \delta$, we have $f(x) > 0$.