- Tutorial problems will be posted every Wednesday, provided there is a tutorial class on the Thursday same week. You are advised to try out the problems before attending tutorial classes, where the questions will be discussed.

- Solutions to tutorial problems will be posted after tutorial classes.

- If you have any questions, please contact Eddie Lam via echlam@math.cuhk.edu.hk or in person during office hours.

1. Prove the following by using $\epsilon - \delta$ definitions, or sequential criterion.

   (a) $\lim_{x \to 1} \frac{x}{1+x} = \frac{1}{2}$.

   (b) $\lim_{x \to 8} \sqrt[8]{x} = 2$.

   (c) $\lim_{x \to 1} x^3 - 2x = -1$.

   (d) $\lim_{x \to 2} \frac{x^3 - 2}{x^2 - 3} = 6$.

   (e) $\lim_{x \to 0} x \cos x (x^2 + 1) = 0$.

   (f) $\lim_{x \to 1} \frac{x-1}{x+1} \neq 1$.

   (g) $\lim_{x \to 0} x + \frac{x}{|x|}$ does not exist.

2. Suppose $c \in \mathbb{R}$, and f is a function on $\mathbb{R}$ so that $\lim_{x \to c} f(x)^2 = L$.

   (a) If $L = 0$, show that $\lim_{x \to c} f(x) = 0$ as well.

   (b) If $L \neq 0$, provide an example in which $\lim_{x \to c} f(x)$ does not exist.

3. Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = x$ if x is rational, and $f(x) = 0$ for x irrational.

   (a) Show that $\lim_{x \to 0} f(x)$ exists.

   (b) Show that $\lim_{x \to c} f(x)$ does not exist for $c \neq 0$.

4. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a bounded function so that $\lim_{x \to c} f(x)$ does not exist, prove that there are two convergent sequence $(x_n), (y_n)$ with $\lim x_n = \lim y_n = c$, so that $(f(x_n)), (f(y_n))$ are convergent sequences with $\lim f(x_n) \neq \lim f(y_n)$.