THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH 2058 Honours Mathematical Analysis I 2022-23
Tutorial 5
13th October 2022

- Tutorial problems will be posted every Wednesday, provided there is a tutorial class on the Thursday same week. You are advised to try out the problems before attending tutorial classes, where the questions will be discussed.
- Solutions to tutorial problems will be posted after tutorial classes.
- If you have any questions, please contact Eddie Lam via echlam@math.cuhk.edu.hk or in person during office hours.

1. For the following subsets, determine whether they are (i) closed, (ii) compact. If the subset is non-compact, exhibit an example of open cover which does not admit a finite subcover.
   (a) $(0, 1]$.
   (b) $[0, \infty)$.
   (c) $\{\frac{1}{n} : n \in \mathbb{N}\}$.
   (d) $\mathbb{Q} \cap [0, 1]$.
2. Prove that if $X \subset \mathbb{R}$ is a compact set, then $\sup X$ exists and $\sup X \in X$.
3. Show that closed subsets of compact subset is again compact.
4. By using the open cover definition of compact subsets, show that the union and intersection of two non-empty compact subsets are again compact.
5. Prove the following generalization of the nested interval theorem: Suppose $\{K_n\}$ is a decreasing sequence of compact subsets in $\mathbb{R}$, i.e. $... \subset K_n \subset K_{n-1} \subset ... \subset K_2 \subset K_1$, then show that $\bigcap_{i=1}^{\infty} K_i$ is nonempty.