THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH 2058 Honours Mathematical Analysis I 2022-23
Homework 7 solutions
9th November 2022

- Homework will be posted on both the course webpage and blackboard every Tuesday. Students are required to upload their solutions on blackboard by 23:59 p.m. next Tuesday. Additional announcement will be made if there are no homework that week.
- Please send an email to echlam@math.cuhk.edu.hk if you have any questions.

1. See Q6 of tutorial 8.
2. Let $S = \{x \in \mathbb{R} : f(x) = 0\}\}$, suppose that $(x_n) \subset S$ is convergent with $\lim x_n = x$, then $f(x_n) = 0$ for any n, therefore by sequential criterion and continuity, $0 = \lim f(x_n) = f(x)$. Therefore $x \in S$.
3. (a) Suppose that f is continuous at $c \in A$, then for any $\epsilon > 0$, there exists $\delta > 0$ so that whenever $x \in B$ and $0 < |x - c| < \delta$, we have $|f(x) - f(c)| < \epsilon$. In particular, if $x \in A \subset B$ satisfying $0 < |x - c| < \delta$, the same conclusion $|f(x) - f(c)| = |g(x) - g(c)| < \epsilon$ would hold, this is equivalent to the continuity of g at c.

   (b) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 1$ for $x \geq 0$ and $f(x) = 0$ for $x < 0$, and $A \subset \mathbb{R}$ be the subset $\{x \in \mathbb{R} | x \geq 0\}\}$. then $g = f|_A$ is continuous at $c = 0$, since g is just the constant function and $\lim_{x \to 0, x \in A} g(x) = \lim_{x \to 0^+} g(x) = 0 = g(0)$. Meanwhile $\lim_{x \to 0} f(x)$ does not exist since the two one-sided limits do not agree.