THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH 2058 Honours Mathematical Analysis I 2022-23
Homework 7
9th November 2022

- Homework will be posted on both the course webpage and blackboard every Tuesday. Students are required to upload their solutions on blackboard by 23:59 p.m. next Thursday. Additional announcement will be made if there are no homework that week.
- Please send an email to echlam@math.cuhk.edu.hk if you have any questions.

1. (P.129 Q7) Let $f : \mathbb{R} \to \mathbb{R}$ be a function that is continuous at c, with $f(c) > 0$, show that there exists a neighborhood $I_\delta(c) = (c - \delta, c + \delta)$ so that for all $x \in I_\delta(c)$ we have $f(x) > 0$.
2. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function, let $S = \{x \in \mathbb{R} | \, f(x) = 0\}$ be the zero set of f. Prove that if (x_n) is a convergent sequence in S, then $\lim x_n =: x \in S$ (i.e. S is a closed subset).
3. Let $A \subset B \subset \mathbb{R}$, let $f : B \to \mathbb{R}$, and g be the restriction of f to A, i.e. $g : A \to \mathbb{R}$ is defined by $g(x) := f(x)$ for any $x \in A$.
   (a) Show that if f is continuous at c, then g is also continuous at c.
   (b) Show by providing a counterexample that may exist a point $c \in A$ so that g is continuous at c but f is not.