

**THE CHINESE UNIVERSITY OF HONG KONG**  
**Department of Mathematics**  
**MATH 2058 Honours Mathematical Analysis I 2022-23**  
**Homework 5**  
**25th October 2022**

- Homework will be posted on both the course webpage and blackboard every Tuesday. Students are required to upload their solutions on blackboard by 23:59 p.m. next Tuesday. Additional announcement will be made if there are no homework that week.
  - Please send an email to [echlam@math.cuhk.edu.hk](mailto:echlam@math.cuhk.edu.hk) if you have any questions.
1. (P.91 Q5) Let  $x_n = \sqrt{n}$ , show that  $\lim |x_{n+1} - x_n| = 0$  but  $x_n$  is not a Cauchy sequence.
  2. (P.91 Q9) Let  $(x_n)$  be a sequence, suppose that there is some  $0 < r < 1$  so that  $|x_{n+1} - x_n| < r^n$  for all  $n \in \mathbb{N}$ , prove that  $(x_n)$  is a Cauchy sequence.
  3. (P.110 Q11) Prove the following limits using  $\epsilon - \delta$  definition.
    - (a)  $\lim_{x \rightarrow 3} \frac{2x+3}{4x-9} = 3$ .
    - (b)  $\lim_{x \rightarrow 6} \frac{x^2-3x}{x+3} = 2$