

**THE CHINESE UNIVERSITY OF HONG KONG**  
**Department of Mathematics**  
**MATH 2058 Honours Mathematical Analysis I 2022-23**  
**Homework 4**  
**11th October 2022**

- Homework will be posted on both the course webpage and blackboard every Tuesday. Students are required to upload their solutions on blackboard by 23:59 p.m. next Thursday. Additional announcement will be made if there are no homework that week.
  - Please send an email to [echlam@math.cuhk.edu.hk](mailto:echlam@math.cuhk.edu.hk) if you have any questions.
1. (P.84 Q5) Let  $(x_n)$  and  $(y_n)$  be two sequences, define  $(z_n)$  to be the "shuffled" sequence, by  $z_1 := x_1, z_2 = y_1, \dots, z_{2n-1} = x_n$  and  $z_{2n} = y_n$ . Prove that  $\lim z_n$  exists if and only if both  $\lim x_n$  and  $\lim y_n$  exists and are equal.
  2. (P.84 Q12) Show that if  $(x_n)$  is unbounded, then there exists a subsequence  $(x_{n_k})$  so that  $\lim 1/x_{n_k} = 0$ .
  3. (P.84 Q14) Let  $(x_n)$  be a bounded sequence, let  $s = \sup\{x_n : n \in \mathbb{N}\}$ , suppose that  $s \notin \{x_n : n \in \mathbb{N}\}$ , show that there is a subsequence  $(x_{n_k})$  so that  $\lim x_{n_k} = s$ .