THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH 2058 Honours Mathematical Analysis I 2022-23
Homework 3
5th October 2022

- Homework will be posted on both the course webpage and blackboard every Tuesday. Students are required to upload their solutions on blackboard by 23:59 p.m. next Thursday. Additional announcement will be made if there are no homework that week.
- Please send an email to echlam@math.cuhk.edu.hk if you have any questions.

1. (P.84 Q4a) Show that the following sequence is divergent: $(x_n) = (1 - (-1)^n + 1/n)$.
2. (P.84 Q6) Let $x_n = n^{\frac{1}{n}}$
   (a) Prove that $x_{n+1} < x_n$ if and only if $(1 + 1/n)^n < n$. Show that this holds for $n \geq 3$. Conclude that (x_n) is eventually decreasing and $x := \lim x_n$ exists.
   (b) Use the fact that $limx_{2n}$ is also equal to x to conclude that $x = 1$.
3. (P.84 Q9) Suppose that every subsequence of (x_n) has a subsequence that converges to 0, prove that $\lim x_n = 0$.