

THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH 2058 Honours Mathematical Analysis I 2022-23
Homework 1
13th September 2022

- Homework will be posted on both the course webpage and blackboard every Tuesday. Students are required to upload their solutions on blackboard by 23:59 p.m. next Tuesday. Additional announcement will be made if there are no homework that week.
- Please send an email to echlam@math.cuhk.edu.hk if you have any questions.

1. (P.44 Q2) Let $S = \{\frac{1}{n} - \frac{1}{m} \mid n, m \in \mathbb{N}\}$, find $\inf S$ and $\sup S$.

2. (P.45 Q8) Let X be a nonempty subset of \mathbb{R} , and let f and g be defined on X and have bounded ranges in \mathbb{R} , show that

$$\sup\{f(x) + g(x) \mid x \in X\} \leq \sup\{f(x) \mid x \in X\} + \sup\{g(x) \mid x \in X\},$$

and

$$\inf\{f(x) \mid x \in X\} + \inf\{g(x) \mid x \in X\} \leq \inf\{f(x) + g(x) \mid x \in X\}.$$

Give examples to show that each of these inequalities can be either equalities or strict inequalities.