

2022-23 MATH2048: Honours Linear Algebra II

Homework 5

Due: 2022-10-14 (Friday) 23:59

For the following homework questions, please give reasons in your solutions. Scan your solutions and submit it via the Blackboard system before due date.

1. Sec. 2.2 Q15
2. Consider a linear transformation $T : V \rightarrow W$. Prove or disprove the following.
 - (a) If T has a right inverse, must it have a left inverse?
 - (b) If T has a left inverse, must it have a right inverse?
 - (c) If T has both a left and a right inverse, must it be invertible? (That is, must the left and right inverse be the same?)
 - (d) If T has a unique right inverse S , is T necessarily invertible? (Hint. Consider $ST + S - I$.)
3. Consider a linear transformation $T : V \rightarrow W$, where $\dim(V) = \dim(W) = n$. Show that if T has a left inverse U , then U is also a right inverse of T , thus T is invertible. (Hint. Sec. 2.4 Q10(b), prove it if you use it)
4. Sec. 2.4 Q20
5. Sec. 2.4 Q24

The following are extra recommended exercises not included in homework.

1. Sec. 2.2 Q13
2. Sec. 2.4 Q17
3. Sec. 2.4 Q19
4. Sec. 2.4 Q21