

**Math 2050, HW 1. Due: 23 Sep 2022**

- (1) Find the supremum and infimum of the set

$$S = \{x \in \mathbb{R} : x < \frac{1}{x}\}.$$

Justify your answer.

- (2) Suppose  $S$  is a non-empty subset of  $\mathbb{R}$  which is bounded from above. Show that  $\sup S = -\inf\{-s : s \in S\}$ .
- (3) Show that if  $A, B$  are bounded subsets of  $\mathbb{R}$ . Show that  $\sup(A + B) = \sup A + \sup B$ , and  $\inf(A + B) = \inf A + \inf B$
- (4) Let  $x > 0$ , show that there is  $n \in \mathbb{N}$  such that  $\frac{1}{2^n} < x$ .
- (5) Let  $x > 0$ , show that there exists a unique  $m \in \mathbb{N}$  such that  $m \leq x < m + 1$ . (In this case, we call such  $m$  as  $[x]$ ).