P119.4 $\int_0^{\pi} e^{(1+i)x}\,dx = \frac{e^{(1+i)x}}{1+i}\Big|_0^{\pi} = \frac{e^{\pi}\cdot e^{i\pi} - e^{0}}{1+i} = \frac{(1-i)(e^{\pi}e^{i\pi}-1)}{(1+i)(1-i)} = \frac{(1-i)(-e^{\pi}-1)}{2}$

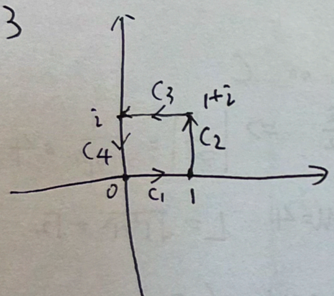
$$= -\frac{1+e^{\pi}}{2} + i\,\frac{1+e^{\pi}}{2}$$

And since $\int_0^{\pi} e^{(1+i)x}\,dx = \int_0^{\pi} e^{x}\cos x\,dx + i\int_0^{\pi} e^{x}\sin x\,dx$

we have $\int_0^{\pi} e^{x}\cos x\,dx = -\frac{1+e^{\pi}}{2}$

$\int_0^{\pi} e^{x}\sin x\,dx = \frac{1+e^{\pi}}{2}$

P133.3



$C_1: z = x$

$C_2: z = 1+iy$

$C_3: z = (1-x)+i$

$C_4: z = i(1-y)$

$C = C_1 + C_2 + C_3 + C_4 \Rightarrow \int_C f\,dz = \int_{C_1} f\,dz + \dots + \int_{C_4} f\,dz$

(1) $\int_{C_1} f(z)\,dz = \int_0^1 \pi e^{\pi x}\,dx = e^{\pi} - 1$

(2) $\int_{C_2} f(z)\,dz = \int_0^1 \pi e^{\pi(1-iy)}\, i\,dy = 2e^{\pi}$

(3) $\int_{C_3} f(z)\,dz = \int_0^1 \pi e^{\pi[(1-x)-i]}\, -1\,dx = e^{\pi} - 1$

(4) $\int_{C_4} f(z)\,dz = \int_0^1 e^{-\pi(1-y)i}\, -i\,dy = -2$

$\therefore \int_C f\,dz = \int_{C_1} f\,dz + \int_{C_2} f\,dz + \int_{C_3} f\,dz + \int_{C_4} f\,dz = 4(e^{\pi}-1)$

P133.4 The path $C = C_1 + C_2$ where $C_1: z = x + ix^3 \quad x \in [-1, 0]$ and $C_2: z = x + ix^3 \quad x \in [0, 1]$

$f(z) = 1$ on C_1 and $f(z) = 4x^3$ on C_2

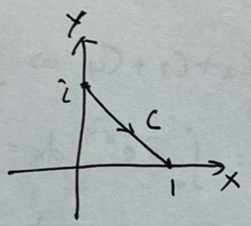
$$\text{Thus } \int_C f(z)\,dz = \int_{C_1} f(z)\,dz + \int_{C_2} f(z)\,dz$$

$$= \int_{-1}^{0} 1 + i3x^2\,dx + \int_0^1 4x^3(1 + i3x^2)\,dx$$

$$= 1 + i + 1 + 2i = 2 + 3i$$

P133.6 $$\int_C e^{(i\log z)}\,dz = \int_0^{\pi} e^{i\cdot i\theta}\, ie^{i\theta}\,d\theta = i\int_0^{\pi} e^{(i-1)\theta}\,d\theta = i\cdot\frac{e^{(i-1)\pi} - 1}{i-1} = \frac{(1+e^{-\pi})(i-1)}{2}$$

P138.2. $f(z) = \frac{1}{z^4}$



Since z lies on C

$$|z| \geq \frac{\sqrt{2}}{2} \Rightarrow \left|\frac{1}{z^4}\right| = \frac{1}{|z|^4} \leq 4$$

$\therefore$ Take $M = 4 \quad L = \sqrt{1^2 + 1^2} = \sqrt{2}$

we have $\left|\int_C \frac{dz}{z^4}\right| \leq ML = 4\sqrt{2}$

P138.5. since z lies on C_R, $\left|\frac{\log z}{z^2}\right| = \frac{|\log R + i\theta|}{R^2} \leq \frac{\pi + \log R}{R^2}$

By taking $M = \frac{\pi + \ln R}{R^2} \quad L = 2\pi R$

we have $\left|\int_{C_R} \frac{\log z}{z^2}\,dz\right| \leq ML = 2\pi\frac{\pi + \ln R}{R}$

And By L'Hospital's Rule, $\lim_{R\to\infty} \frac{\pi + \ln R}{R} = \lim_{R\to\infty} \frac{1/R}{1} = 0$

$$\lim_{R\to\infty} \int_{C_R} \frac{\log z}{z^2}\,dz = 0$$

P147.5 Since the brunch is not defined at $z=-1$

we replace the integrand by another brunch

$$z^i = \exp(i\log z) \qquad -\frac{\pi}{2} < \arg z < \frac{3\pi}{2}$$

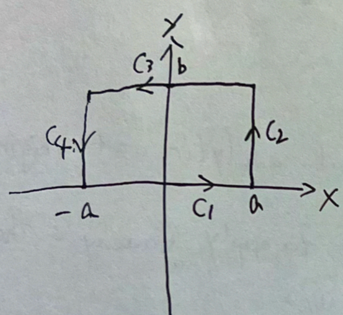
since they agree along C.

we have $\int_{-1}^{1} z^i\,dz = \frac{z^{i+1}}{1+i}\Big|_{-1}^{1} = \frac{1}{i+1}\left[e^{(i+1)\log 1} - e^{(i+1)(\log(-1))}\right]$

$$= \frac{1}{1+i}(1-e^{-\pi}e^{i\pi}) = \frac{1+e^{-\pi}}{1+i}$$

$$= \frac{(1+e^{-\pi})(1-i)}{2}$$

P159.4. a)



$$\int_{C_1} e^{-z^2}dz = \int_{-a}^{a} e^{-x^2}dx = 2\int_0^a e^{-x^2}dx$$

$$\int_{C_3} e^{-z^2}dz = -\int_{-a}^{a} e^{-(x+bi)^2}dx$$

$$= -e^{b^2}\int_{-a}^{a} e^{-x^2}e^{-i2bx}dx$$

$$= -e^{b^2}\int_{-a}^{a} e^{-x^2}\cos 2bx\,dx + \underbrace{ie^{b^2}\int_{-a}^{a} e^{-x^2}\sin 2bx\,dx}_{\text{odd function}} \;=0$$

$$= -e^{b^2}\int_{-a}^{a} e^{-x^2}\cos 2bx\,dx$$

C_1: $z=x$

C_2: $z=a+iy$

C_3: $z=x+bi$

C_4: $z=-a+bi-iy$

$$\int_{C_2} e^{-z^2}dz = \int_0^b e^{-(a+iy)^2}\,i\,dy = ie^{-a^2}\int_0^b e^{y^2}e^{-i2ay}dy$$

$$\int_{C_4} e^{-z^2}dz = -\int_0^b e^{-(-a+iy)^2}\,i\,dy = -ie^{-a^2}\int_0^b e^{y^2}e^{2iay}dy$$

Thus By Cauchy's theorem

$$2\int_0^a e^{-x^2}dx - 2e^{b^2}\int_0^a e^{-x^2}\cos 2bx\,dx + ie^{-a^2}\int_0^b e^{y^2}e^{-i2ay}dy - ie^{-a^2}\int_0^b e^{y^2}e^{i2ay}dy = 0$$

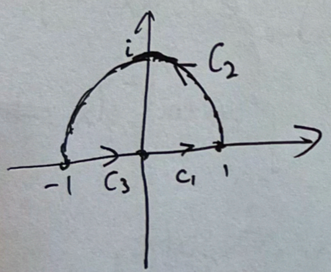
$$\Rightarrow \int_0^a e^{-x^2}\cos 2bx\,dx = e^{-b^2}\int_0^a e^{-x^2}dx + e^{-(a^2+b^2)}\int_0^b e^{y^2}\sin 2ay\,dy$$

b) since $\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$, if we let $a \to \infty$ in part (a)

and $\left|e^{-(a^2+b^2)}\int_0^b e^{y^2}\sin 2ay\,dy\right| \le e^{-(a^2+b^2)}\int_0^b e^{y^2}dy \to 0$ as $a \to \infty$

$$\Rightarrow \int_0^\infty e^{-x^2}\cos 2bx\,dx = \frac{\sqrt{\pi}}{2}e^{-b^2}$$

Prbl. 6.



since f is not analytic at origin we are not able to apply Cauchy's theorem

$f(z) = \sqrt{r}\,e^{i\theta/2}$

$C_1: z = re^{i\theta}$ with $\theta = 0$ $r \in [0,1]$

$C_2: z = e^{i\theta}$ $\theta \in [0,\pi]$

$C_3: z = r$ $r \in [-1,0]$

$$\therefore \int_{C_1} f(z)dz = \int_0^1 \sqrt{r}\,dr = \left[\frac{2}{3}r^{3/2}\right]_0^1 = \frac{2}{3}$$

$$\int_{C_2} f(z)dz = \int_0^\pi e^{i\theta/2}\cdot ie^{i\theta}\,d\theta = \frac{2}{3}(-i-1)$$

$$\int_{C_3} f(z)dz = -\int_0^1 \sqrt{r}\,e^{i\pi/2}(-1)dr = \frac{2}{3}i$$

$$\therefore \int_C f(z)dz = 0$$