

THE CHINESE UNIVERSITY OF HONG KONG

Department of Mathematics

MATH4010 Functional Analysis 2022-23 Term 1

Homework 6

Deadline: 2022-10-27 Thursday

Notice:

- All the assignments must be submitted before the deadline.
- Each assignment should include your name and student ID number.

1. If X and Y are Banach spaces and $T_n: X \rightarrow Y$, $n = 1, 2, \dots$ a sequence of bounded linear operators, show that the following statements are equivalent:
 - (a) the sequence $(\|T_n\|)$ is bounded,
 - (b) the sequence $(\|T_n x\|)$ is bounded for every $x \in X$,
 - (c) the sequence $(|f(T_n x)|)$ is bounded for every $x \in X$ and every $f \in Y^*$.
2. Let X and Y be normed spaces and $T: X \rightarrow Y$ be a closed linear operator (that is, the graph of T is closed in $X \times Y$).
 - (a) Show that the image of a compact subset of X is closed in Y .
 - (b) Show that the inverse image of a compact subset of Y is closed in X .

— *THE END* —