THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH4010 Functional Analysis 2022-23 Term 1
Homework 1
Deadline: 2022-09-22 Thursday

Notice:
- All the assignments must be submitted before the deadline.
- Each assignment should include your name and student ID number.

1. Let $B(S)$ be the vector space of all bounded $\mathbf{F}$-valued functions on a nonempty set S. Define

$$\|x\| = \sup\{|x(t)| : t \in S\}.$$

Prove that $(B(S), \|\cdot\|)$ is a complete normed space (cf. [Textbook, Theorem 3.5]).

2. Show that for $1 \leq p < \infty$,

$$c_{00} \subset \ell_p \subset c_0 \subset c \subset \ell_\infty$$

and all inclusions are proper. (Please refer to [Textbook, Section 3.3 & 3.4] for the definitions of above spaces.)

Also show that

(a) c_{00} is dense in the space c_0,

(b) c_{00} is dense in ℓ_p,

(c) c_{00} is not dense in c,

(d) c_{00} is not dense in ℓ_∞,

in the topology defined by the sup-norm, $\|\cdot\|_\infty$.

Note. The textbook, *Ovchinnikov, Functional analysis, Springer*, can be downloaded from the library web.

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