MATH3060 HW8 Due date: No need to hand-in

1. Show that any norm on $\mathbb{R}^n$ is equivalent to the usual Euclidean norm $\|\cdot\|_2$ defined by $\|x\|_2 = \sqrt{\sum_{i=1}^{n} x_i^2}$ for $x=(x_1,\cdots,x_n) \in \mathbb{R}^n$.

2. Let $\ell_2 = \{ \{x_n\}_{n=1}^{\infty} : \sum_{n=1}^{\infty} x_n^2 < \infty \}$ with metric

$$d_2(\{x_n\},\{y_n\}) = \sqrt{\sum_{n=1}^{\infty} (x_n - y_n)^2}.$$

Show that $H = \{ \{x_n\}_{n=1}^{\infty} \in \ell_2 : |x_n| \leq \frac{1}{n} \}$ is nowhere dense in (ℓ_2, d_2).

3. Show that $\mathcal{P} = \{ p \in C[0,1] : p \text{ is a polynomial} \}$ is a set of 1^{st} category.

4. Show that

$$Z = \{ f \in M[0,1] : f([0,1]) \text{ is nowhere dense in } \mathbb{R} \}$$

is a residual set of $(M[0,1], d_\infty)$, where $M[0,1]$ is the space of bounded functions on $[0,1]$, and d_∞ is the metric induced from supnorm $\|\cdot\|_\infty$.

(End)