MATH3060 HW5 Due date: Nov 4, 2022 (at 11:00 am)

1. Let $a>0$, define a mapping $T: C[-a,a] \to C[-a,a]$ by

$$Tx(t) = 1 + \int_0^t s\, x(s)\,ds.$$

Let $x(t) \equiv 1$ on $[-a,a]$. Find $T^n x$, $\forall n \geq 0$. Does $\{T^n x\}$ converge in $(C[-a,a], d_\infty)$? If so, what is the limit?

2. Show that the equation $\cos x - 2x^4 - x = 1.001$ has a solution near $x=0$.

3. Let $\Phi: \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $\Phi\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \sin x - 2y^4 \\ \sin y + x^2 \end{pmatrix}$.

Show that $\Phi\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0.01 \end{pmatrix}$ has a solution.

4. Let $K(x,t) \in C([0,1]\times[0,1])$. Show that there exists $\lambda > 0$ such that for all $g \in C[0,1]$, there exists a unique solution $y \in C[0,1]$ of the integral equation $$y(x) = g(x) + \lambda \int_0^1 K(x,t)\, y(t)\,dt.$$

(End)