MATH2050
Tutorial 9

November 13, 2022

Abstract

This tutorial discusses about the concept of continuity and uniformity inside continuous functions on potentially compact intervals.

Some exercises are taken from exercises of Bartle section 5.3, 4, you should work on more exercise there.

1. Prove that any powers, expoential, compositions of continuous functions are continuous.

2. Prove that values of continuous functions is determined entirely by its value on any dense set (e.g. $\mathbb{Q}$). Prove equivalently that if two continuosu functions values agree on any dense set, then they are the same.

3. If f is continuous functions that is additive (i.e. $f(x) + f(y) = f(x + y)$ for any $x, y \in \mathbb{R}$), then $f(x) = f(1) \cdot x$.

4. Prove that locally bounded functions are bounded uniformly on bounded and closed interval.

5. If f is continuous and I is any interval, then $f(I)$ is also an interval. What if f is discontinuous only at a point there?

6. If f is continuous function and take only irrational values, can it be non-constant?

7. ~~If f is uniformly continuous on $\mathbb{R}_{\geq 0}$, then f is always bounded above by some linear function (i.e. at most $O(x)$).~~

8. ~~Suppose you are to prove a statement: "If f is locally bounded but uniformly, then it is bounded above by some linear function.". How can you define "bounded but uniformly" to make this valid?~~