

P.3

Question time: Q on lectures.

## Part 2: Exercises & Examples

To answer Q1, we need show by different algebraic, order, completeness properties of  $\mathbb{R}$  that such axioms reconstruct the  $\mathbb{R}$  in our usual intuition.

1. Show  $(a+b)^2 = a^2 + 2ab + b^2$ ,  
 $(-a)^2 = a^2$ .

2. show that  $1 < 2 < 3 < \dots$  and  $(n, n+1) \cap \mathbb{N} = \emptyset$ .  
 $\forall n \in \mathbb{N}$ .

3. Do dictation on:

A. definition of limit of sequence, boundedness of sequence.

B. axioms of real numbers.

C. completeness and Archimedean properties.

4. (standard e.g.)

show that  $\sup(X) = -\inf(-X)$ . for  $X \subseteq \mathbb{R}$   
 $X \neq \emptyset$ .

(can you make it more formal by introducing  $\epsilon$ os?)

5. For definition of limit of sequences, change  $\epsilon$  by  $10\epsilon$ ,  $\delta$  by  $\frac{\delta}{5}$ , prove that the definitions are equivalent.