

1. Rewrite each of the following sentences as suggested/indicated:

(i) 若天氣好, 我去行山 (If weather is good then I go hiking)

(a) Rephrase the above sentence by using "whenever"

(ii) 凡中大學生, 是好人

(a) $\dots \subseteq \dots$, where the sets are defined by

(b) If \dots then (用(a)定义之集合)

(c) $\dots \in \dots$ for $\dots \in \dots$

(d) \dots , whenever \dots

2. State the negation for each of (i), and (ii) of Q1.

3. Use axiom I show that, $\forall a, b \in \mathbb{R}$,
 $(a+b)^2 = a^2 + 2ab + b^2$, and $(a+b)^3 = a^3 + \dots$
 and $(-a)^2 = a^2$ other properties in I & II

4. * Show^{by} MI (or extended MI) that

$$1 < 2 < 3 < \dots$$

and that

$$(n, n+1) \cap \mathbb{N} = \emptyset \quad \forall n \in \mathbb{N}$$

Can you extend the results to \mathbb{Z} , integers? ?

5. Fill the blanks: call \bar{x}

a largest element of a set X of real numbers if _____ and _____

(用「最大元」及「上确界」)

Show that, when exists, such an element is unique (& henceforth one uses the standard notation $\max X$)

5. Do Q5 for smallest ele.

6. What is meant that

(i) $\bar{x} \in \mathbb{R}$ is not a largest ele. of X

(ii) \underline{x} is not a smallest ele. of X

7. A set Y of real numbers

is said to be bounded above

if there exists a real number n

such that

$$\dots \leq n \dots$$

Show that this is the case iff $\exists n \in \mathbb{N}$ satisfying the above ineq.

Fill the blanks and state the

negation (what is meant that

Y is not bounded above)

7. Do Q7 for bounded below.

8*. Provide a bounded
(= bounded below and bounded
above) set^X of real numbers
such that $\min X$, $\max X$
do not exist. Check your
assertion.

9* (i) Show that $(\forall x \in \mathbb{R})$
 $x, -x \leq |x|$

and that $x = |x|$ or $-x = |x|$.

(ii) Let $x, y \in \mathbb{R}$ and $0 < \alpha \in \mathbb{R}$.
Show that

$$|x| < \alpha \Leftrightarrow -\alpha < x < \alpha$$

$$|x-y| < \alpha \Leftrightarrow x-\alpha < y < x+\alpha$$

(The assumption $\alpha > 0$ is redundant)

10. Show, $\forall a, b \in \mathbb{R}$, that

$$|a \cdot b| = |a| \cdot |b|$$

and that

$$||a| - |b|| \leq |a \pm b| \leq |a| + |b|$$

(^{or "reverse"} "anti"-triangle inequality + triangle inequality)
反三角不等式

11* We sometimes write

(the notation suggested by looking at the

graphs of $x \mapsto \max\{f(x), g(x)\}$

$x \mapsto \min\{f(x), g(x)\}$

for real-valued functions f, g).

$$a \vee b := \max\{a, b\} \quad \forall a, b \in \mathbb{R}.$$

$$a \wedge b := \min\{a, b\}$$

Show that, $\forall a, b \in \mathbb{R}$,

$$-(a \vee b) = (-a) \wedge (-b), \quad -(a \wedge b) = (-a) \vee (-b)$$

$$a \vee b = \frac{a+b+|a-b|}{2} \quad (\frac{\text{和} + \text{差}}{2} = \text{大数})$$

$$a \wedge b = \frac{(a+b) - |a-b|}{2} \quad (\frac{\text{和} - \text{差}}{2} = \text{小数})$$

小学学过?

12. Let $\emptyset \neq B \subseteq \mathbb{R}$ and

$$-B := \{-b : b \in B\}.$$

Show that

(i) B is bounded below iff (if and only if) $-B$ is bounded above

(ii) $l \in \mathbb{R}$ is a l.b of B iff $-l$ is an u.b of $-B$

13. Let $\emptyset \neq X \subseteq \mathbb{R}$. Show that
(by I, II, III)

$$-\sup X = \inf(-X)$$

whenever X is bounded above or $-X$ is bounded below.

(More details are in Hw 2).