THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH 6022B (Second term, 2022-23)
Essay and Presentation

The course assessment of MATH 6022B will consist of one in-class presentations (which everyone must attend) and a written mathematical essay on selected topics. Note the following important dates:

- *In-class presentation*: **April 24, 2023**, 2:30PM-5:15PM
- *Essay submission deadline (by email)*: **May 6, 2023** at 11:59PM

Presentations and Essay

Presentations: Every student should give his/her own presentation in English. The presentations will be evaluated in terms of content, clarity and pedagogy. You should give some motivation and introductory/background material about the chosen topic, highlight some of the most interesting theorems related, and give some connections to other areas if possible. Each presentation should be roughly 20-30 minutes long.

Written essay: You should write up in LaTeX a mathematical article about the your selected topic. The essay should be at least 5 pages long. Each student is responsible for writing up his/her own essay and send it to the instructor via email by the deadline stated above. More specifically, the essay should comprise of precise statements of the problems you have studied, some of the background of the mathematics involved, and at least a sketch of the ideas of the proofs of some major theorems. References should be supplied at the end of the report (which does not count towards the 5-page requirement).

Some suggested topics

You are welcomed to choose a topic of your own to work on. Below are some suggestions:

- In class we talked about curve shortening flow in $\mathbb{R}^2$ but it is also interesting to study curve shortening flow in higher codimensions (i.e. inside $\mathbb{R}^N$ for $N \geq 3$). It was first systematically studied by Altschuler "Singularities of the curve shrinking flow for space curves", J. Differential Geom. (1991) and Altschuler-Grayson, "Shortening space curves and flow through singularities", J. Differential Geom. (1992).
- Study in detail the local version of Huisken's monotonicity formula due to Ecker, "A local monotonicity formula for mean curvature flow", Annals of Math (2001)
- It is instructive from the PDE point of view to study the evolution of entire graphs under mean curvature flow which often possess better long-time behaviour. See e.g. Ecker and Huisken, "Mean curvature flow of entire graphs", Annals of Math (1989); Ecker and Huisken, "Interior estimates for hypersurfaces moving by mean curvature", Invent. Math. (1991)
- Colding and Minicozzi studied the regularity of the level set flow for mean-convex hypersurfaces. They proved that even though the arrival time may fail to be in C^2, it is always twice differentiable everywhere with uniform bounded second derivative, and smooth away from its critical points. See, e.g., Colding and Minicozzi, "Differentiability of the arrival time", Comm. Pure Appl. Math (2016)

- The classification theory of non-collapsed ancient solutions to mean curvature flow by Choi and Brendle, "Uniqueness of convex ancient solutions to mean curvature flow in $\mathbb{R}^3$", Invent. Math. (2019); "Uniqueness of convex ancient solutions to mean curvature flow in higher dimensions", Geom. & Topo. (2021)
- Colding and Minicozzi introduced a notion of "entropy" whichis defined by the supremum over all Gaussian integrals with varying centers and scales, thus invariant under rigid motions and dilations. It measures geometric complexity of mean curvature flow. There are some recent progress on conjectures of Colding–Ilmanen–Minicozzi–White concerning the sharp lower bound on entropy for hypersurfaces. See e.g. Colding and Minicozzi, "Generic mean curvature flow I: generic singularities", Annals of Math. (2012); Lu Wang, "Entropy in mean curvature flow", ICM proceedings

Useful Links

- The Not So Short Introduction to LaTeX (https://tobi.oetiker.ch/lshort/lshort.pdf)
- MathSciNet (https://mathscinet.ams.org/mathscinet/)