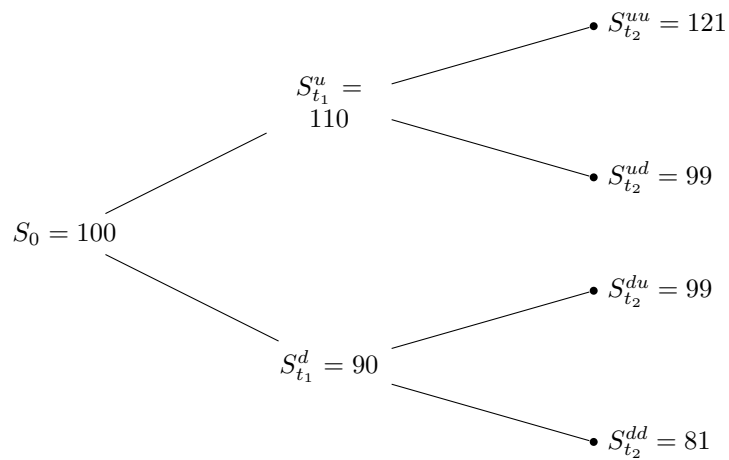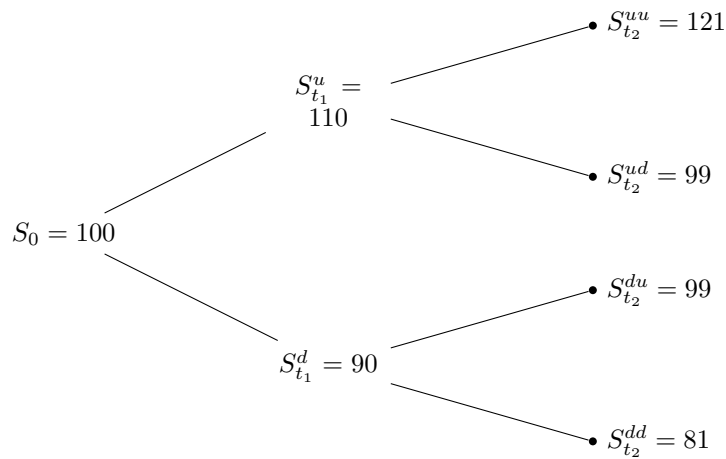# Math 4210 Assignment 1

Due date: October 10th, 2022, 11:59pm

1. Consider a two step binomial tree model for an **American put option** whose strike price is $K = 101$, with the following parameters: $S_{t_0} = 100$, $u = 1.1$, $d = 0.9$, $1 + r\Delta t = 1.02$. Find the no-arbitrage price of the option at time zero and describe/give the replication strategy. *Note:* For an American put option, its holder can exercise the option at any time of t_0, t_1, t_2, and the payoff of the option at the exercise time t_k is given by $(K - S_{t_k})_+$.

$$S_0 = 100$$

$$S_{t_1}^u = 110$$

$$S_{t_1}^d = 90$$

$$S_{t_2}^{uu} = 121$$

$$S_{t_2}^{ud} = 99$$

$$S_{t_2}^{du} = 99$$

$$S_{t_2}^{dd} = 81$$

2. Consider again the two step binomial tree model with the same parameters as in problem 1: $S_{t_0} = 100$, $u = 1.1$, $d = 0.9$, $1 + r\Delta t = 1.02$. This time, please find the no-arbitrage price at time zero and the replication strategy for a **lookback call option**, assuming that it can only be exercised at maturity. *Note:* A Lookback call option is an option whose strike price equals to the minimum price experienced by the underlying asset during the life of the call, i.e. the payoff at maturity time $T = t_2 = 2\Delta t$ is given by $(S_{t_2} - M_{t_2})_+$, with $M_{t_2} = \min(S_{t_0}, S_{t_1}, S_{t_2})$.

$$S_0 = 100$$

$$S^u_{t_1} = 110$$

$$S^d_{t_1} = 90$$

$$S^{uu}_{t_2} = 121$$

$$S^{ud}_{t_2} = 99$$

$$S^{du}_{t_2} = 99$$

$$S^{dd}_{t_2} = 81$$

3. Finally, we consider a two step binomial tree model for an **Asian call option** where the averaging A_{t_2} is done over all three prices observed, i.e., $A_{t_2} = (S_{t_0} + S_{t_1} + S_{t_2})/3$, with the following (different than previous!) parameters: $S_{t_0} = 100$, $u = 1.05$, $d = 0.95$, $1 + r\Delta t = 1.02$. Suppose that the option can only be exercised at maturity $T = t_2$ and the payoff is given by $(A_{t_2} - K)_+$ with $K = 100$. Find the no-arbitrage price and the replication strategy for this Asian call option.

$$S_{t_1}^u = 105$$
$$S_0 = 100$$
$$S_{t_1}^d = 95$$
$$S_{t_2}^{uu} = 110.25$$
$$S_{t_2}^{ud} = 99.75$$
$$S_{t_2}^{du} = 99.75$$
$$S_{t_2}^{dd} = 90.25$$