An EEG Blind Source Separation Algorithm Based On A Weak Exclusion Principle

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***Abstract*— The question of how to separate individual brain and non-brain signals, mixed by volume conduction in electroencephalographic (EEG) and other electrophysiological recordings, is a significant problem in contemporary neuroscience. This study proposes and evaluates a novel EEG Blind Source Separation (BSS) algorithm based on a weak exclusion principle (WEP). The chief point in which it differs from most previous EEG BSS algorithms is that the proposed algorithm is not based upon the hypothesis that the sources are statistically independent. Our first step was to investigate algorithm performance on simulated signals which have ground truth. The purpose of this simulation is to illustrate the proposed algorithm's efficacy. The results show that the proposed algorithm has good separation performance. Then, we used the proposed algorithm to separate real EEG signals from a memory study using a revised version of Sternberg Task. The results show that the proposed algorithm can effectively separate the non-brain and brain sources.**

I. INTRODUCTION

Blind Source Separation (BSS) is the separation of a set of sources from mixed observations, with very little prior knowledge of the sources or their mixing coefficients[1]. BSS extracts vital information from the data while simultaneously highlighting the underlying forces which drive observed phenomena[2]. The cocktail party problem is a famous example of this[3]. Useful applications are possible in a variety of fields, including speech, images, telecommunications, and biomedical signal processing[4].

Electroencephalography (EEG) non-invasively measures voltage fluctuations resulting from ionic current within the neurons of the brain with very high temporal resolution. Because all the electrodes are placed along the scalp, the observed EEG data actually is a mixture of all the active sources. BSS methods have been shown to be very useful tools to extract these sources from EEG[5]. The transmission delay between electrode and source is negligible. Furthermore, each measured signal can be plausibly assumed to be a linear mixture of source signals, given that the electrical signals must travel through human tissue to reach the electrodes[6]. The unmixing matrix's inverse can also be used to provide a spatial illustration of each BSS-extracted signal's associated scalp location[7].

Here we should point out that scalp-recorded EEG signals also include non-brain sources; these include electroculographic (EOG) and electromyographic (EMG) activities. These are linearly mixed with brain sources at the scalp electrodes. EEG BSS is also capable of separating these kinds of signals[8].

Among BSS's distinct branches is Independent Component Analysis (ICA)[9]. At its core, ICA represents an attempt to identify linear representations of non-gaussian data in an effort to render its components statistically independent. For this reason, various ICA methods are usually based on various measures of non-gaussianity. There are some other useful methods for source separation; these include: Principal Component Analysis (PCA), Factor Analysis (FA) and Sparse Component Analysis (SCA)[10][11][12]. In a perusal of the published literature, it is evident that the hypothesis of statistical independence forms the basis of most BSS algorithms. Much fewer are based on deterministic hypotheses.

In this article, we propose a novel EEG BSS algorithm based on a deterministic principle, which we call it weak exclusion principle (WEP). Due to the difficulties in assessing ground truth of EEG, we have used simulated signals which have "ground truth" to test the algorithm's performance. Afterwards, we also perform the proposed algorithm on actual EEG data to separate the non-brain and brain sources.

The paper is organized as follows: First, an introduction to the problem of EEG blind source separation; second, a formulation of the problem and the proposed mathematical model. Finally, Sections 3 and 4 detail the "Simulated data" and the"EEG experimental data," followed by the conclusion.

II. THE PROPOSED MATHEMATICAL MODEL

The problem of EEG blind source separation problem is formulated as follows: Stacking Mixtures in an $N \times K$ matrix $\mathbf{X}$ (N: time samples, K: the number of electrodes).This is the EEG signal we observe. Then, we try to unmix them as a linear combination of K sources. Mathematically, we would like to find the $N \times K$ matrix $\mathbf{S}$ describing the time-samples of the sources and the mixing $K \times K$ matrix $\mathbf{A}$ such that

$$\mathbf{X} = \mathbf{SA}; \text{ or, more visually, } \underbrace{\overbrace{\boxed{\mathbf{X}}}^{K}}_{N} = \underbrace{\overbrace{\boxed{\mathbf{S}}}^{K}}_{N} \times \underbrace{\overbrace{\boxed{\mathbf{A}}}^{K}}_{K} \quad (1)$$

In reality, $\mathbf{X}$ is a noisy linear mixture of source signals. Thus, there is another term for noise: $\mathbf{X} = \mathbf{SA}+\mathbf{N}$. However the noise term will not be included in the derivation for simplicity.

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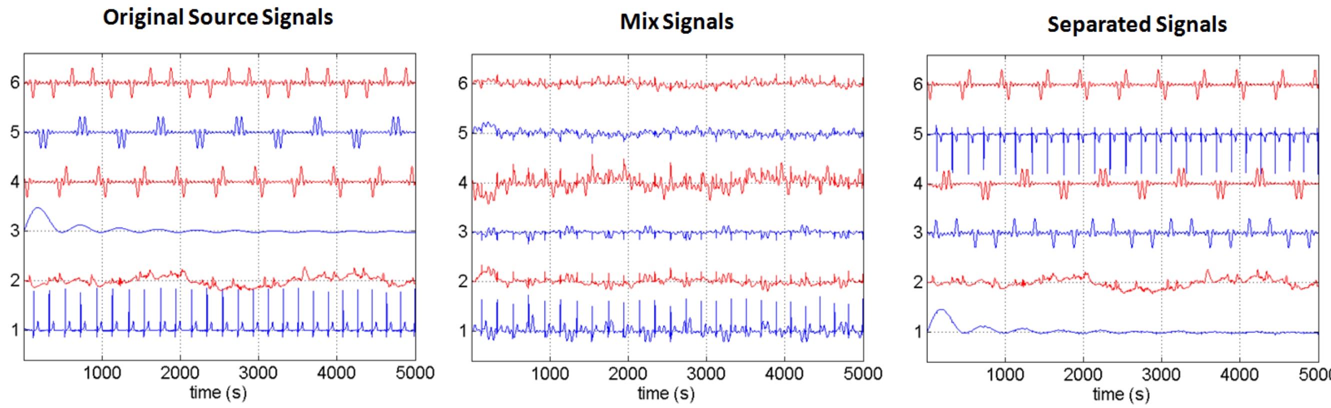

Fig. 1. Illustration of Blind Source Separation based on Weak Exclusion Principle (WEP) Using Simulated Data.

A. Exclusion Principle (EP)

An $N \times N$ matrix $\mathbf{S}$ is exclusive, *iff* for all row index $i \in [1,...N], \mathbf{S}_{ij} \neq 0 \Longrightarrow \mathbf{S}_{ij'} = 0, for\ \forall j' \in [1,...K] \setminus j$. The exclusive principle can equivalently be expressed as follows: For any $N \times N$ diagonal matrix $\mathbf{W}$, there exists a diagonal $K \times K$ matrix $\mathbf{D_W}$ such that

$$\mathbf{S}^{\mathrm{T}}\mathbf{W}\mathbf{S} = \mathbf{D_W} \tag{2}$$

B. Source Retrieval Under The Exclusive Principle

We assume that $\mathbf{X} = \mathbf{SA}$ with unknown $\mathbf{S}$ and $\mathbf{A}$, and that $\mathbf{S}$ satisfies the EP above. The goal is to separate the source matrix $\mathbf{S}$ and, the mixing matrix $\mathbf{A}$, from the mixed matrix $\mathbf{X}$.

From (1), we can find two diagonal matrices $\mathbf{W}_1$,$\mathbf{W}_2$, such that

$$\mathbf{S}^{\mathrm{T}}\mathbf{W}_1\mathbf{S} = \mathbf{D}_1 \quad \text{and} \quad \mathbf{S}^{\mathrm{T}}\mathbf{W}_2\mathbf{S} = \mathbf{D}_2 \tag{3}$$

Hence,

$$\mathbf{X}^{\mathrm{T}}\mathbf{W}_1\mathbf{X} = \mathbf{A}^{\mathrm{T}}\mathbf{D}_1\mathbf{A} \quad \text{and} \quad \mathbf{X}^{\mathrm{T}}\mathbf{W}_2\mathbf{X} = \mathbf{A}^{\mathrm{T}}\mathbf{D}_2\mathbf{A} \tag{4}$$

Which leads to

$$\underbrace{\left(\mathbf{X}^{\mathrm{T}}\mathbf{W}_2\mathbf{X}\right)^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{W}_1\mathbf{X}}_{Y} = \mathbf{A}^{-1} \underbrace{\mathbf{D}_2^{-1}\mathbf{D}_1}_{\text{D: diagonal}} \mathbf{A} \tag{5}$$

This equation shows that the column of $\mathbf{A}^{-1}$ are eigenvectors of the known matrix $\mathbf{Y}$, with eigenvalues given by diagonal elements of $\mathbf{D}$.

The choice of the matrix $\mathbf{W}_1$,$\mathbf{W}_2$ is very important for the separation. Here, we name them the "Separation Matrix".

Following is the procedure of EEG blind source separation algorithm based on EP:

1) Compute the matrix $\mathbf{Y}$.
2) Compute the eigenvalue decomposition of $\mathbf{Y} = \mathbf{VDV}^{-1}$
3) Identify the matrix of eigenvectors as the matrix inverse of the mixing matrix $\mathbf{A}$.

C. Weak Exclusion Principle (WEP)

If the EEG source signals satisfy the EP strictly, at each time instance, there is only one source activated. However, in the proposed algorithm, we use a weaker condition, a weak exclusion principle. That is to say, at each time instance, the EEG signal is dominated by one source which is larger than the others. The other sources don't have to be non-activated at all. This assumption is more consistent with the reality.

Mathematically, this is equivalent to solving a maximization problem:

1) Consider the diagonal matrix $\mathbf{W}_k$ where $\mathbf{W}_k[m,m] = 1$ if $|\mathbf{S}(m,k)| > |\mathbf{S}(m,l)|$ for $l \neq k$.
2) If we know $\mathbf{W}_k$, then an approach to infer $\mathbf{S}(:,k)$ from $\mathbf{X}$ is to find the linear combination u_k of columns of $\mathbf{X}$ that maximizes $\| \mathbf{W}_k\mathbf{X}u_k \|$, under the constraint that the estimated source $\mathbf{S}'_k = \mathbf{X}u_k$ is normalized to 1: $\| \mathbf{X}u_k \| = 1$.
Mathematical Solution: $\mathbf{X}^T\mathbf{W}_k\mathbf{X}u_k = \lambda_k\mathbf{X}^T\mathbf{X}u_k$, where λ_k is the largest eigenvalue of the matrix $(\mathbf{X}^T\mathbf{X})^{-1}(\mathbf{X}^T\mathbf{W}_k\mathbf{X})$.
3) The estimated source is then $\mathbf{X}\underbrace{u_1u_2...u_k}_{\mathbf{U}} = \mathbf{S}'$.
4) So that $\mathbf{A}' = \mathbf{U}^{-1}$.

III. Blind Source Separation Using Simulated Data

Because of EEG lack of "ground truth," we have used simulated signals which have "ground truth" to test the algorithm's performance. The simulated source signal we use is provided by ICALAB[14], which simulates a combination of typical biosignals. The mixing matrices are random matrices. Fig. 1 shows one example of the original source signals, mix signals and the separated result (via our algorithm).

According to ICALAB, some ICA algorithms have failed to separate the sources which have smooth bell-shape. When comparing the separated result to the "ground truth," our algorithm's efficacy is readily apparent.

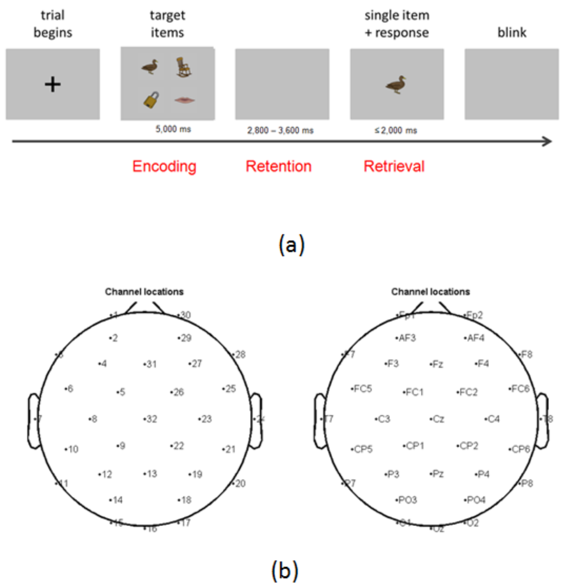

Fig. 2. (a) Illustration of the revised version of Sternberg Task. (b) Channel Locations of the 32-channel ActiveTwo EEG system.

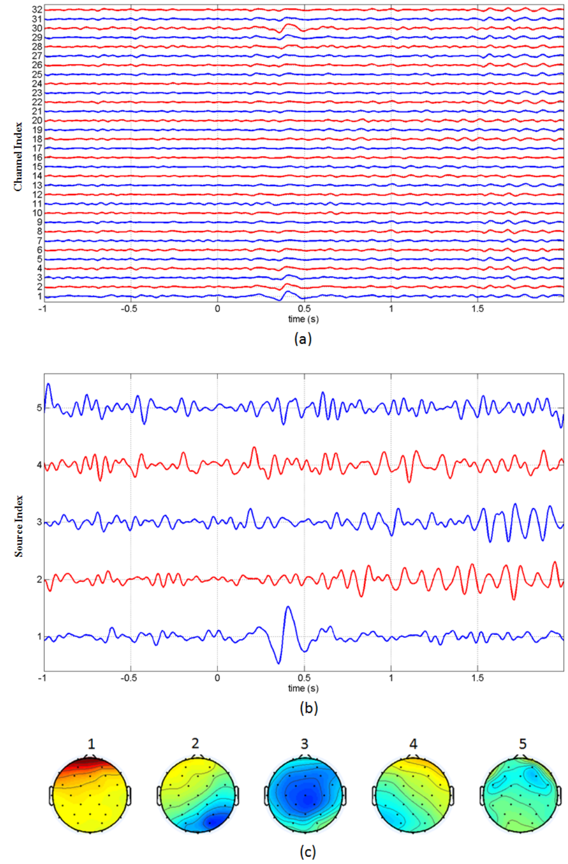

Fig. 3. Illustration of (a)Electrode plots of one single-trial raw EEG recordings, around 0.4s there was a clear eye artifact. (b) The time course of the separated source and (c) The topographies of the separated source.

IV. Blind Source Separation Using EEG Experimental Data

We also tested the algorithm on actual EEG data. The data used came from a memory study which used a revised version of Sternberg Task. In total, three procedures were involved. First was the "Encoding" step, in which four images appeared on the screen, and subjects were asked to memorize them. Second was "Retention", in which subjects were asked to keep the memory of the images from the screen. Third was "Retrieval," in which one item appeared on the screen and the subjects were asked to recall whether or not this item was part of the previous set. Each experiment has 64 trials. The illustration of the paradigm is shown in Fig.2. EEG recordings were originally acquired using 32-channel ActiveTwo EEG system (BioSemi B.V., Amsterdam, The Netherlands) with a sampling rate of 1024 Hz. The data we used for separation was from one female subject (Age: 64). The EEG signal was down-sampled to 200Hz and band-pass filtered from 4 Hz to 20 Hz. We performed an eigenvalue decomposition first on the EEG data and kept the components that can represent 90% variance to reduce dimensions.This procedure decides the source number.

A. Non-brain Sources

To illustrate that our WEP BSS algorithm can effectively separate non-brain sources, we used one single-trial retention-procedure EEG with clear eye artifacts entering the WEP BSS algorithm. Fig.3 shows the EEG recording and the source separation results. Here it is necessary to note that the sources are ranked in a descending order. That is to say, the first source represents the largest variance in this dataset. We have to clarify because by the nature of the BSS algorithm, the scale of the topographies is not the same as the real signal scale. However, it can reflect landscapes of the separated sources. From the EEG recordings, we can see around 0.4s there was a clear eye artifact. From the separated sources' time course, we can see the first source had a clear peak around the same time interval. Besides, the spatial illustration of this source shows a clear peak around the eye area. Fig.4 shows the reconstructed EEG signal by removing the source related to eye artifacts and the source separation results based on this reconstructed signal. We can clearly see the eye artifacts have been removed.

B. Brain Sources

We averaged the data across trials and used our WEP BSS to analyze each of these three procedures separately. The time course and topographies of the separated sources are shown in Fig. 5. From the separation results we can see that the first two procedures have five main sources and the last one has four main sources. In the encoding period, the second source shows a strong activity in the occipital lobe and the time course of this source shows that its peak is around 200ms-300ms after the onset of the presentation of the four images. This may relate to the well-known visual evoked potential (VEP). Besides, we can observe the second source has activities around the prefrontal area and the bilateral occipital area. Previous studies have prove that mental processes that require subvocal rehearsal

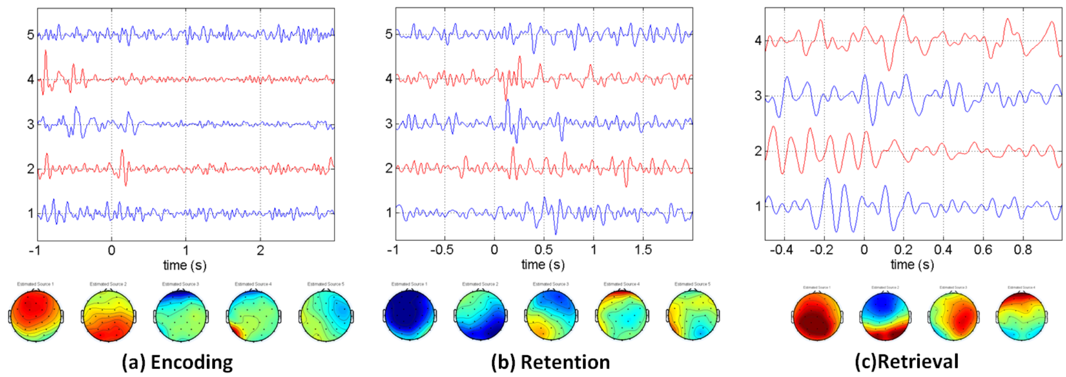

Fig. 5. The time course and topographies of the separated sources of three different procedures.(a) Encoding (b) Retention (c) Retrieval.

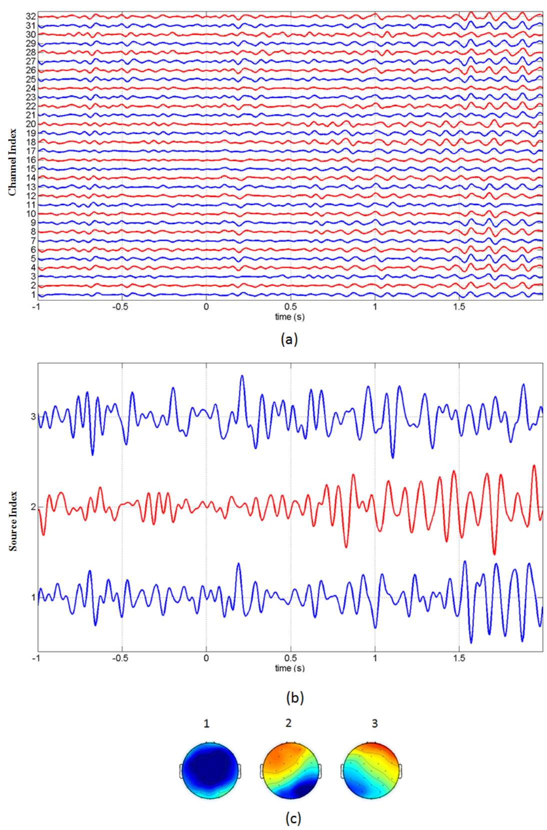

Fig. 4. Illustration of the (a)Electrode plots of the reconstructed EEG signal by removing the source related to eye artifacts. (b) The time course of the separated source and (c) The Topographies of the separated source.

preferentially activated the left prefrontal cortex, the bilateral occipital cortex[15,16].

V. Conclusions

The proposed EEG BSS algorithm, based on a weak exclusion principle (WEP), may represent an appropriate technique to solve the EEG source separation problem. The results show that the proposed algorithm can effectively separate the non-brain and brain sources. In the future, this algorithm should be tested on larger EEG datasets, providing further confirmation of real world applicability.

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