

CENG 3420

Computer Organization & Design



Lecture 05: Logic Basis

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(Textbook: Chapter 2.4)

Spring 2023



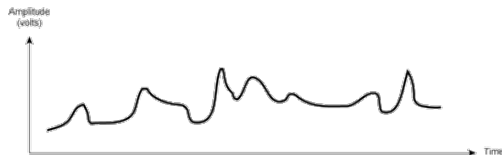
Numeral System

Analog vs. Digital



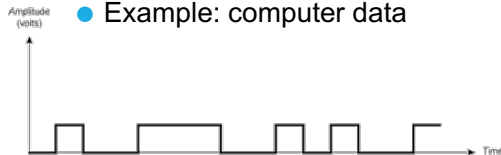
Analog Signal

- Vary in a smooth way over time
- Analog data are continuous valued
 - Example: audio, video



Digital Signal

- Maintains a constant level then changes to another constant level (generally operate in one of the two states)
- Digital data are discrete valued
 - Example: computer data





Number Systems

- An ordered set of symbols, called digits, with relations defined for addition, subtraction, multiplication, and division
- **Radix** or **base** of the number system is the total number of digits allowed in the number system
- Commonly used numeral systems

System Name	Decimal	Binary	Octal	Hexadecimal
Radix	10	2	8	16
First seventeen positive integers	0	0	0	0
	1	1	1	1
	2	10	2	2
	3	11	3	3
	4	100	4	4
	5	101	5	5
	6	110	6	6
	7	111	7	7
	8	1000	10	8
	9	1001	11	9
	10	1010	12	A
	11	1011	13	B
	12	1100	14	C
	13	1101	15	D
	14	1110	16	E
	15	1111	17	F
	16	10000	20	10

- In the 2009 film Avatar, Na'vi race employs an **octal** numeral system.





Conversion from Decimal Integer

- ❑ Step 1: Divide the decimal number by the radix (number base)
- ❑ Step 2: Save the remainder (first remainder is the least significant digit)
- ❑ Repeat steps 1 and 2 until the quotient is zero
- ❑ Result is in reverse order of remainders



EX: L02-1

- EX1: Convert 36_8 to binary value
- EX2: Convert 36_{10} to binary value



Unsigned Binary Representation

Hex	Binary	Decimal
0x00000000	0...0000	0
0x00000001	0...0001	1
0x00000002	0...0010	2
0x00000003	0...0011	3
0x00000004	0...0100	4
0x00000005	0...0101	5
0x00000006	0...0110	6
0x00000007	0...0111	7
0x00000008	0...1000	8
0x00000009	0...1001	9
	...	
0xFFFFFFFFC	1...1100	$2^{32} - 4$
0xFFFFFFFDD	1...1101	$2^{32} - 3$
0xFFFFFFFDE	1...1110	$2^{32} - 2$
0xFFFFFFFFF	1...1111	$2^{32} - 1$

2^{31} 2^{30} 2^{29} ... 2^3 2^2 2^1 2^0 bit weight

31 30 29 ... 3 2 1 0 bit position

1 1 1 ... 1 1 1 1 bit



1 0 0 0 ... 0 0 0 0 - 1



$2^{32} - 1$



Signed Binary Representation

	2'sc binary	decimal
$-2^3 =$	1000	-8
$-(2^3 - 1) =$	1001	-7
	1010	-6
	1011	-5
	1100	-4
	1101	-3
	1110	-2
	1111	-1
	0000	0
	0001	1
	0010	2
	0011	3
	0100	4
	0101	5
	0110	6
	0111	7

complement all the bits

0101 1011

and add a 1 and add a 1

0110 1010

complement all the bits

$2^3 - 1 =$



EX: L02-2

- For an n-bit signed binary numeral system, what's the largest positive number and the smallest negative number?



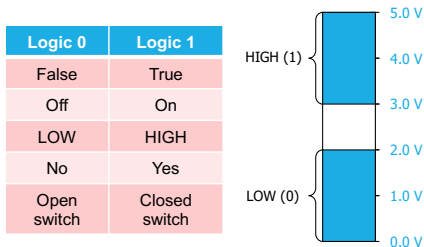
Digital Signal Representation

□ Active HIGH

- High voltage means On

□ Active LOW

- Low voltage means On





- Just like in grade school (carry/borrow 1s)

$$\begin{array}{r} 0111 \\ + 0110 \\ \hline \end{array}$$

$$\begin{array}{r} 0111 \\ - 0110 \\ \hline \end{array}$$

$$\begin{array}{r} 0110 \\ - 0101 \\ \hline \end{array}$$

- Two's complement operations are easy: do subtraction by negating and then adding

$$\begin{array}{r} 0111 \\ - 0110 \\ \hline \end{array} \quad \rightarrow \quad \begin{array}{r} 0111 \\ + 1010 \\ \hline \end{array}$$

- Overflow (result too large for finite computer word). E.g., adding two n-bit numbers does not yield an n-bit number

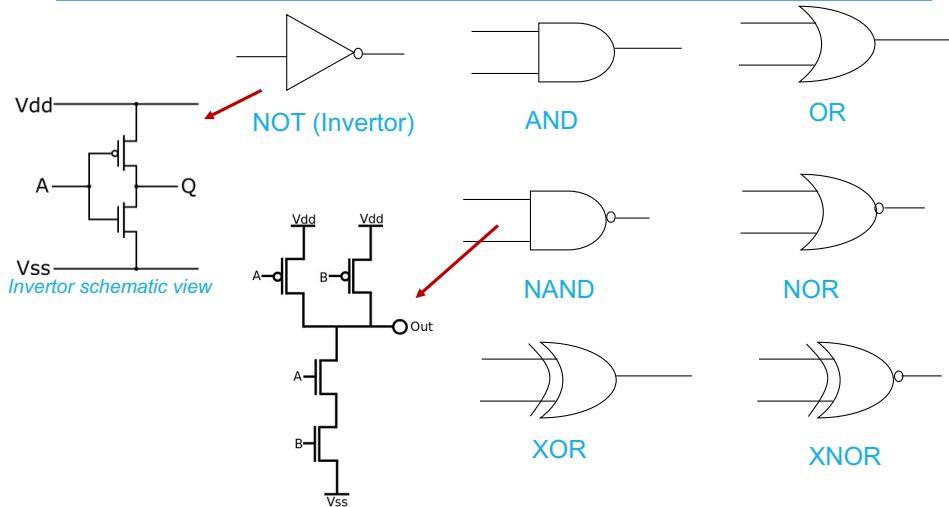
$$\begin{array}{r} 0111 \\ + 0001 \\ \hline \end{array}$$



Logic Gates



Logic Gates



❑ What is the schematic view of an AND gate?

EX: L02-3

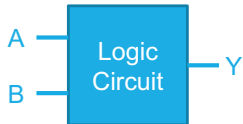


- Please draw NOR gate schematic view



Truth Table

- A means for describing how a logic circuit's output depends on the logic levels present at the circuit's inputs
- The number of input combinations will equal 2^N for an N-input truth table



Inputs		Output
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1



EX: L02-4

Determine the true table of a three-input AND gate