CMSC 5743

Efficient Computing of Deep Neural Networks

Lecture 06: Quantization

Bei Yu
CSE Department, CUHK
byu@cse.cuhk.edu.hk

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These slides contain/adapt materials developed by

- Hardware for Machine Learning, Shao Spring 2020 @ UCB
- 8-bit Inference with TensorRT
- Junru Wu et al. (2018). "Deep *k*-Means: Re-training and parameter sharing with harder cluster assignments for compressing deep convolutions". In: *Proc. ICML*
- Shijin Zhang et al. (2016). "Cambricon-x: An accelerator for sparse neural networks". In: *Proc. MICRO*. IEEE, pp. 1–12
- Jorge Albericio et al. (2016). "Cnvlutin: Ineffectual-neuron-free deep neural network computing". In: *ACM SIGARCH Computer Architecture News* 44.3, pp. 1–13

Scientific Notation

Decimal representation

mantissa

exponent

$6.02_{10} \times 10^{23}$

decimal point

radix (base)

- Normalized form: no leadings 0s (exactly one digit to left of decimal point)
- Alternatives to representing 1/1,000,000,000
  - Normalized: 1.0×10^{-9}
  - Not normalized: $0.1 \times 10^{-8}, 10.0 \times 10^{-10}$

Scientific Notation

Binary representation

mantissa

exponent

$$1.01_{two} \times 2^{-1}$$

"binary point"

radix (base)

- Computer arithmetic that supports it called floating point, because it represents numbers where the binary point is not fixed, as it is for integers

Normalized Form

- Floating Point Numbers can have multiple forms, e.g.

$$
\begin{aligned}
0.232 \times 10^4 &= 2.32 \times 10^3 \\
&= 23.2 \times 10^2 \\
&= 2320. \times 10^0 \\
&= 232000. \times 10^{-2}
\end{aligned}
$$

- It is desirable for each number to have a unique representation => Normalized Form
- We normalize Mantissa's in the Range $[1..R)$, where R is the Base, e.g.:
  - $[1..2)$ for BINARY
  - $[1..10)$ for DECIMAL

Floating-Point Representation

- Normal format: $+1.xxx\ldots x_{two} * 2^{yyy\ldots y_{two}}$

31	30 ... 23	22 ... 0
S	Exponent	Significand
1 bit	8 bits	23 bits

- **S represents Sign**
- **Exponent represents y's**
- **Significand represents x's**
- **Represent numbers as small as 2.0×10^{-38} to as large as 2.0×10^{38}**

Floating-Point Representation (FP32)

- IEEE 754 Floating Point Standard
  - Called **Biased Notation**, where bias is number subtracted to get real number
  - IEEE 754 uses bias of 127 for single prec.
  - Subtract 127 from Exponent field to get actual value for exponent
  - 1023 is bias for double precision

- Summary (single precision, or fp32):

31	30 … 23	22 … 0
S	Exponent	Significand
1 bit	8 bits	23 bits

- $(-1)^S \times (1 + \text{Significand}) \times 2^{(\text{Exponent}-127)}$

Floating-Point Representation (FP16)

- IEEE 754 Floating Point Standard
  - Called **Biased Notation**, where bias is number subtracted to get real number
  - IEEE 754 uses bias of 15 for half prec.
  - Subtract 15 from Exponent field to get actual value for exponent

- Summary (half precision, or fp15):

15	15 10	9 0
S	**Exponent**	**Significand**
1 bit	**5 bits**	**10 bits**

- $(-1)^S \times (1 + \text{Significand}) \times 2^{(\text{Exponent}-15)}$

Question:

What is the IEEE single precision number $40C0\ 0000_{16}$ in decimal?

Question:

What is the IEEE single precision number 40C0 0000_{16} in decimal?

- Binary: 0100 0000 1100 0000 0000 0000 0000 0000
- Sign: +
- Exponent: 129 - 127 = +2
- Mantissa: 1.100 0000 $\ldots_2 \rightarrow 1.5_{10} \times 2^{+2}$
- $\rightarrow$ +110.0000 $\ldots_2$
- Decimal Answer = $+6.0_{10}$

Question:

What is -0.5_{10} in IEEE single precision binary floating point format?

Question:

What is -0.5_{10} in IEEE single precision binary floating point format?

- Binary: 1.0... $\times 2^{-1}$ (in binary)
- Exponent: 127 + (-1) = 01111110
- Sign bit: 1
- Mantissa: 1.000 0000 0000 0000 0000 0000
- Binary representation: 1011 1111 0000 0000 0000 0000 0000 0000

- Integers with a binary point and a bias
  - "slope and bias": y = s*x + z
  - Qm.n: m (# of integer bits) n (# of fractional bits)

s = 1, z = 0

2^2	2^1	2^0	Val
0	0	0	**0**
0	0	1	**1**
0	1	0	**2**
0	1	1	**3**
1	0	0	**4**
1	0	1	**5**
1	1	0	**6**
1	1	1	**7**

s = 1/4, z = 0

2^0	2^-1	2^-2	Val
0	0	0	**0**
0	0	1	**1/4**
0	1	0	**2/4**
0	1	1	**3/4**
1	0	0	**1**
1	0	1	**5/4**
1	1	0	**6/4**
1	1	1	**7/4**

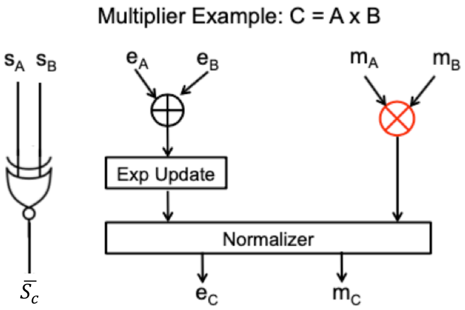
s = 4, z = 0

2^4	2^3	2^2	Val
0	0	0	**0**
0	0	1	**4**
0	1	0	**8**
0	1	1	**12**
1	0	0	**16**
1	0	1	**20**
1	1	0	**24**
1	1	1	**28**

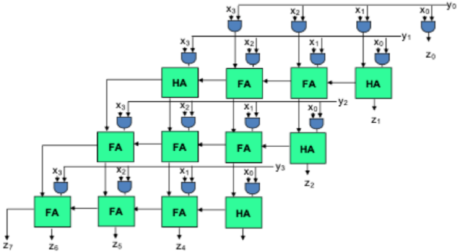
s = 1.5, z =10

2^2	2^1	2^0	Val
0	0	0	**1.5*0 +10**
0	0	1	**1.5*1 +10**
0	1	0	**1.5*2 +10**
0	1	1	**1.5*3 +10**
1	0	0	**1.5*4 +10**
1	0	1	**1.5*5 +10**
1	1	0	**1.5*6 +10**
1	1	1	**1.5*7 +10**

Multipliers



Floating-point multiplier



Fixed-point multiplier

Linear quantization

Representation:

Tensor Values = FP32 scale factor * int8 array + FP32 bias

Do we really need bias?

Two matrices:

```
A = scale_A * QA + bias_A
B = scale_B * QB + bias_B
```

Let's multiply those 2 matrices:

```
A * B = scale_A * scale_B * QA * QB +
        scale_A * QA * bias_B       +
        scale_B * QB * bias_A       +
        bias_A  * bias_B
```

Do we really need bias?

Two matrices:

A = scale_A * QA + ~~bias_A~~
B = scale_B * QB + ~~bias_B~~

Let's multiply those 2 matrices:

A * B = scale_A * scale_B * QA * QB +
~~scale_A * QA * bias_B +~~
~~scale_B * QB * bias_A +~~
~~bias_A * bias_B~~

Do we really need bias? No!

Two matrices:

```
A = scale_A * QA
B = scale_B * QB
```

Let’s multiply those 2 matrices:

```
A * B = scale_A * scale_B * QA * QB
```

Symmetric linear quantization

Representation:

Tensor Values = FP32 scale factor * int8 array

One FP32 scale factor for the entire int8 tensor

Q: How do we set scale factor?

MINIMUM QUANTIZED VALUE

- Integer range is not completely symmetric. E.g. in 8bit, [-128, 127]
  - If use [-127, 127], $s = \frac{127}{\alpha}$
    - Range is symmetric
    - 1/256 of int8 range is not used. 1/16 of int4 range is not used
  - If use full range [-128, 127], $s = \frac{128}{\alpha}$
    - Values should be quantized to 128 will be clipped to 127
    - Asymmetric range may introduce bias

EXAMPLE OF QUANTIZATION BIAS

Bias introduced when int values are in [-128, 127]

$$A = [-2.2 \quad -1.1 \quad 1.1 \quad 2.2]\ ,\ B = \begin{bmatrix} 0.5 \\ 0.3 \\ 0.3 \\ 0.5 \end{bmatrix}\ ,\ AB = 0$$

8bit scale quantization, use [-128, 127]. s_A=128/2.2, s_B=128/0.5

$$[-128 \quad -64 \quad 64 \quad 127] * \begin{bmatrix} 127 \\ 77 \\ 77 \\ 127 \end{bmatrix} = -127$$

Dequantize -127 will get -0.00853. A small bias is introduced towards -∞

EXAMPLE OF QUANTIZATION BIAS

No bias when int values are in [-127, 127]

$$A = \begin{bmatrix} -2.2 & -1.1 & 1.1 & 2.2 \end{bmatrix},\ B = \begin{bmatrix} 0.5 \\ 0.3 \\ 0.3 \\ 0.5 \end{bmatrix},\ AB = 0$$

8-bit scale quantization, use [-127, 127]. s_A=127/2.2, s_B=127/0.5

$$\begin{bmatrix} -127 & -64 & 64 & 127 \end{bmatrix} * \begin{bmatrix} 127 \\ 76 \\ 76 \\ 127 \end{bmatrix} = 0$$

Dequantize 0 will get 0

MATRIX MULTIPLY EXAMPLE

Scale Quantization

$$\begin{pmatrix} -1.54 & 0.22 \\ -0.26 & 0.65 \end{pmatrix} * \begin{pmatrix} 0.35 \\ -0.51 \end{pmatrix} = \begin{pmatrix} -0.651 \\ -0.423 \end{pmatrix}$$

MATRIX MULTIPLY EXAMPLE

Scale Quantization

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8bit quantization

choose [-2, 2] fp range (scale 127/2=63.5) for first matrix and [-1, 1] fp range (scale = 127/1=127) for the second

$$\begin{pmatrix} -98 & 14 \\ -17 & 41 \end{pmatrix} * \begin{pmatrix} 44 \\ -65 \end{pmatrix} = \begin{pmatrix} -5222 \\ -3413 \end{pmatrix}$$

MATRIX MULTIPLY EXAMPLE

Scale Quantization

$$\begin{pmatrix} -1.54 & 0.22 \\ -0.26 & 0.65 \end{pmatrix} * \begin{pmatrix} 0.35 \\ -0.51 \end{pmatrix} = \begin{pmatrix} -0.651 \\ -0.423 \end{pmatrix}$$

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$$\begin{pmatrix} -98 & 14 \\ -17 & 41 \end{pmatrix} * \begin{pmatrix} 44 \\ -65 \end{pmatrix} = \begin{pmatrix} -5222 \\ -3413 \end{pmatrix}$$

The result has an overall scale of 63.5*127 . We can *dequantize* back to float

$$\begin{pmatrix} -5222 \\ -3413 \end{pmatrix} * \frac{1}{63.5 * 127} = \begin{pmatrix} -0.648 \\ -0.423 \end{pmatrix}$$

REQUANTIZE

Scale Quantization

$$\begin{pmatrix} -1.54 & 0.22 \\ -0.26 & 0.65 \end{pmatrix} * \begin{pmatrix} 0.35 \\ -0.51 \end{pmatrix} = \begin{pmatrix} -0.651 \\ -0.423 \end{pmatrix}$$

8bit quantization

choose [-2, 2] fp range for first matrix and [-1, 1] fp range for the second

$$\begin{pmatrix} -98 & 14 \\ -17 & 41 \end{pmatrix} * \begin{pmatrix} 44 \\ -65 \end{pmatrix} = \begin{pmatrix} -5222 \\ -3413 \end{pmatrix}$$

Requantize output to a different quantized representation with fp range [-3, 3]:

$$\begin{pmatrix} -5222 \\ -3413 \end{pmatrix} * \frac{127/3}{63.5 * 127} = \begin{pmatrix} -27 \\ -18 \end{pmatrix}$$

Greedy Layer-wise Quantization[1]

Quantization flow

- For a fixed-point number, it representation is:

$$n = \sum_{i=0}^{bw-1} B_i \cdot 2^{-f_l} \cdot 2^i,$$

where bw is the bit width and f_l is the fractional length which is dynamic for different layers and feature map sets while static in one layer.

- Weight quantization: find the optimal f_l for weights:

$$f_l = \arg\min_{f_l} \sum |W_{float} - W(bw, f_l)|,$$

where W is a weight and $W(bw, f_l)$ represents the fixed-point format of W under the given bw and f_l.

[1] Jiantao Qiu et al. (2016). "Going deeper with embedded fpga platform for convolutional neural network". In: *Proc. FPGA*, pp. 26–35.

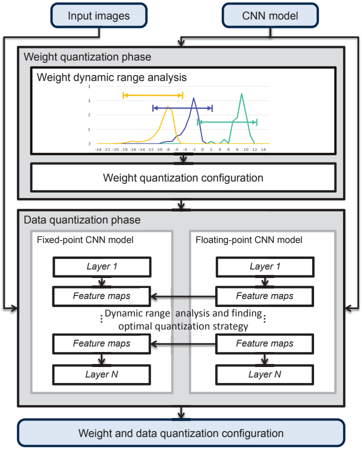
Greedy Layer-wise Quantization

Quantization flow

- Feature quantization: find the optimal f_l for features:

$$f_l = \arg\min_{f_l} \sum |x^+_{float} - x^+(bw, f_l)|,$$

where x^+ represents the result of a layer when we denote the computation of a layer as $x^+ = A \cdot x$.

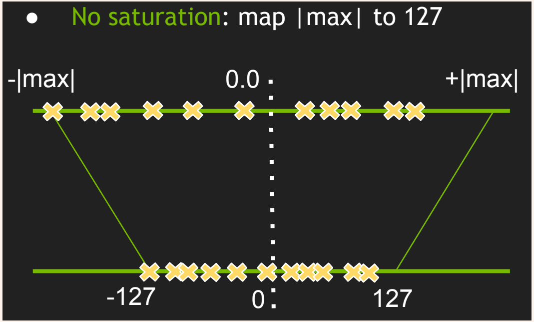


Dynamic-Precision Data Quantization Results

Network	VGG16						
Data Bits	Single-float	16	16	8	8	8	8
Weight Bits	Single-float	16	8	8	8	8	8 or 4
Data Precision	N/A	2^{-2}	2^{-2}	Impossible	$2^{-5}/2^{-1}$	Dynamic	Dynamic
Weight Precision	N/A	2^{-15}	2^{-7}	Impossible	2^{-7}	Dynamic	Dynamic
Top-1 Accuracy	68.1%	68.0%	53.0%	Impossible	28.2%	66.6%	67.0%
Top-5 Accuracy	88.0%	87.9%	76.6%	Impossible	49.7%	87.4%	87.6%

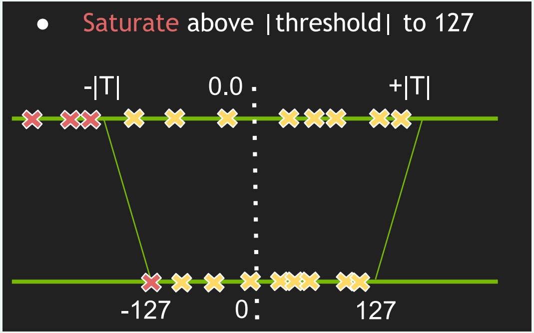
Network	CaffeNet			VGG16-SVD		
Data Bits	Single-float	16	8	Single-float	16	8
Weight Bits	Single-float	16	8	Single-float	16	8 or 4
Data Precision	N/A	Dynamic	Dynamic	N/A	Dynamic	Dynamic
Weight Precision	N/A	Dynamic	Dynamic	N/A	Dynamic	Dynamic
Top-1 Accuracy	53.9%	53.9%	53.0%	68.0%	64.6%	64.1%
Top-5 Accuracy	77.7%	77.1%	76.6%	88.0%	86.7%	86.3%

No Saturation Quantization – INT8 Inference



- Map the maximum value to 127, with unifrom step length.
- Suffer from outliers.

Saturation Quantization – INT8 Inference



- Set a threshold as the maxiumum value.
- Divide the value domain into 2048 groups.
- Traverse all the possible thresholds to find the best one with minimum KL divergence.

Relative Entropy of two encodings

- INT8 model encodes the same information as the original FP32 model.
- Minimize the loss of information.
- Loss of information is measured by Kullback-Leibler divergence (*a.k.a.*, relative entropy or information divergence).
  - P, Q - two discrete probability distributions:

$$D_{KL}(P \| Q) = \sum_{i=1}^{N} P(x_i) \log \frac{P(x_i)}{Q(x_i)}$$

- Intuition: KL divergence measures the amount of information lost when approximating a given encoding.

Straight-Through Estimator (STE)[2]

- A straight-through estimator is a way of estimating gradients for a threshold operation in a neural network.
- The threshold could be as simple as the following function:

$$f(x) = \begin{cases} 1, & x \geq 0 \\ 0, & \text{else} \end{cases}$$

- The derivate of this threshold function will be 0 and during back-propagation, the network will learn anything since it gets 0 gradients and the weights won't get updated.

[2] Yoshua Bengio, Nicholas Léonard, and Aaron Courville (2013). "Estimating or propagating gradients through stochastic neurons for conditional computation". In: *arXiv preprint arXiv:1308.3432*.

PArameterized Clipping acTivation Function (PACT)[3]

- A new activation quantization scheme in which the activation function has a parameterized clipping level α.
- The clipping level is dynamically adjusted vias stochastic gradient descent (SGD)-based training with the goal of minimizing the quantization error.
- In PACT, the convolutional ReLU activation function in CNN is replaced with:

$$f(x) = 0.5\left(|x| - |x-\alpha| + \alpha\right) = \begin{cases} 0, & x \in (\infty, 0) \\ x, & x \in [0, \alpha) \\ \alpha, & x \in [\alpha, +\infty) \end{cases}$$

where α limits the dynamic range of activation to $[0, \alpha]$.

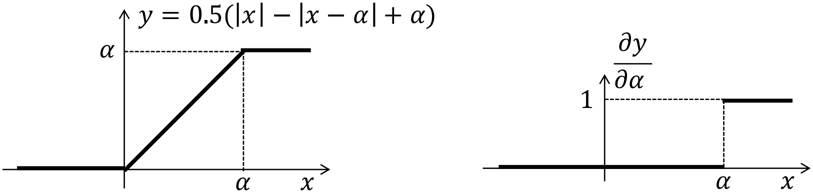
[3] Jungwook Choi et al. (2019). "Accurate and efficient 2-bit quantized neural networks". In: *Proceedings of Machine Learning and Systems* 1.

PArameterized Clipping acTivation Function (PACT)

- The truncated activation output is the linearly quantized to k-bits for the dot-product computations:

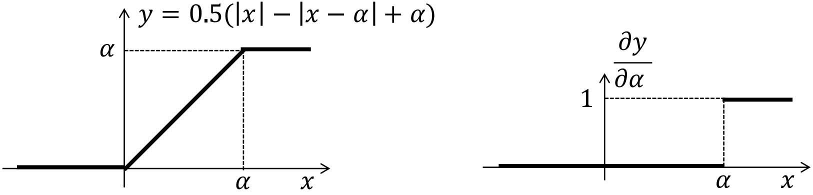
$$y_q = \text{round}\left(y \cdot \frac{2^k - 1}{\alpha}\right) \cdot \frac{\alpha}{2^k - 1}$$

- With this new activation function, α is a variable in the loss function, whose value can be optimized during training.
- For back-propagation, gradient $\frac{\partial y_q}{\partial \alpha}$ can be computed using STE to estimate $\frac{\partial y_q}{\partial y}$ as 1.



PACT activation function and its gradient.

Is Straight-Through Estimator (STE) the best?



PACT activation function and its gradient.

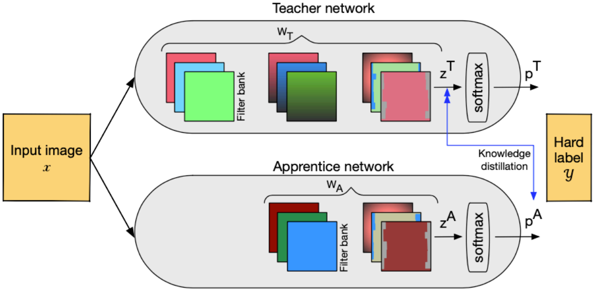
- Gradient mismatch: the gradients of the weights are not generated using the value of weights, but rather its quantized value.
- Poor gradient: STE fails at investigating better gradients for quantization training.

Knowledge Distillation-Based Quantization[4]

- Knowledge distillation trains a student model under the supervision of a well trained teacher model.
- Regard the pre-trained FP32 model as the teacher model and the quantized models as the student models.

$$\mathcal{L}(x; W_T, W_A) = \alpha\mathcal{H}(y, p^T) + \beta\mathcal{H}(y, p^A) + \gamma\mathcal{H}(z^T, p^A) \quad (1)$$

where, W_T and W_A are the parameters of the teacher and the student (apprentice) network, respectively, y is the ground truth, $\mathcal{H}(\cdot)$ denotes a loss function and, α, β and γ are weighting factors to prioritize the output of a certain loss function over the other.



[4] Asit Mishra and Debbie Marr (2017). "Apprentice: Using knowledge distillation techniques to

Further Reading List

- Darryl Lin, Sachin Talathi, and Sreekanth Annapureddy (2016). “Fixed point quantization of deep convolutional networks”. In: *Proc. ICML*, pp. 2849–2858
- Soroosh Khoram and Jing Li (2018). “Adaptive quantization of neural networks”. In: *Proc. ICLR*
- Jan Achterhold et al. (2018). “Variational network quantization”. In: *Proc. ICLR*
- Antonio Polino, Razvan Pascanu, and Dan Alistarh (2018). “Model compression via distillation and quantization”. In: *arXiv preprint arXiv:1802.05668*
- Yue Yu, Jiaxiang Wu, and Longbo Huang (2019). “Double quantization for communication-efficient distributed optimization”. In: *Proc. NIPS*, pp. 4438–4449
- Markus Nagel et al. (2019). “Data-free quantization through weight equalization and bias correction”. In: *Proc. ICCV*, pp. 1325–1334