CENG4480 Homework 2

Due: Nov. 03, 2020

Solutions

Q1 (15%) The circuit shown in Figure 1 represents an n-bit weighted digital-to-analog converter. Each switch S_i is controlled by the corresponding bit of the digital number D_i if the bit is 1 the switch is up; if the bit is 0 the switch is down. Please answer the following two questions:

(1) Determine an expression relating V_0 to the binary input bits D_i and V_{ref}.

(2) If $n = 8$, $V_{ref} = -10V$ and input digits$(D_{n-1}...D_0)_2$ is 32_{10}, please caulate V_0.

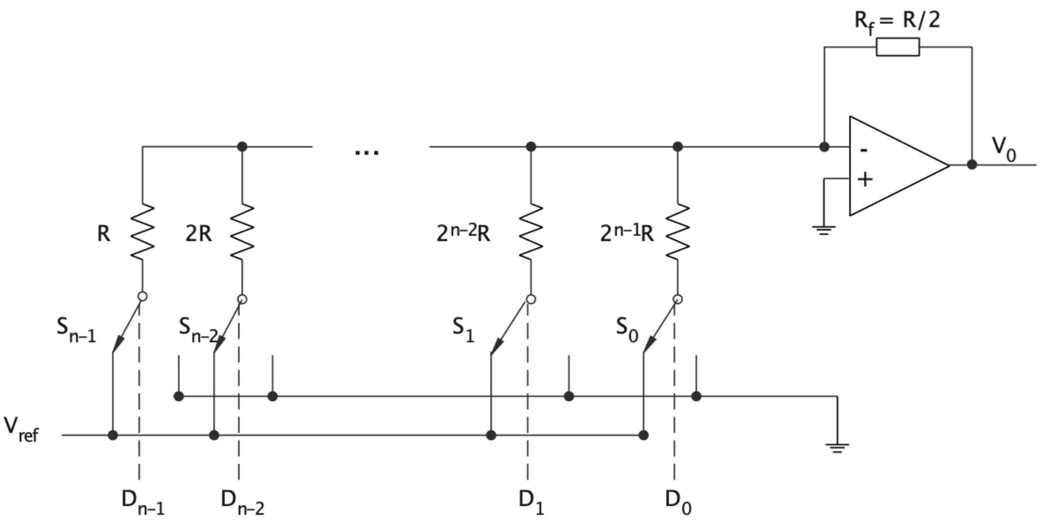

Figure 1: n-bit DAC.

A1 (1)

$$V_0 = \frac{V_{ref}}{2^n} \sum_{i=0}^{n-1} (D_i 2^i) \tag{1}$$

(2)

$$V_0 = \frac{10}{256} 32 = 1.25V \tag{2}$$

Q2 (10%) For R-2R DAC showed in Figure 2, please calculate V_1.

A2

$$V_0 = \frac{V_{ref}}{2^n}(D_n...D_0)_2 = \frac{5V}{2^4}(0101)_2 = \frac{5V}{16}5 = 1.5625V \tag{3}$$

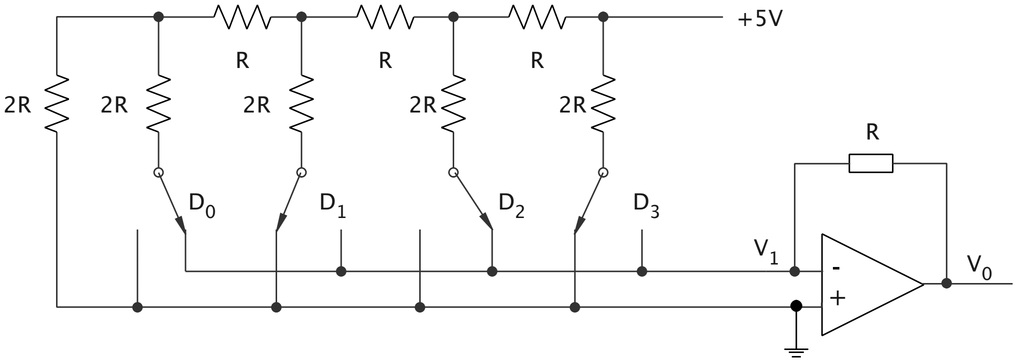

Figure 2: R-2R DAC.

Q3 (15%) A 8-bit analog-to-digital converter (ADC) has the analog input voltage ranging from 0 to +10V. Please answer the following questions.

(1) When the input voltage is 4.48V, calculate the binary output.

(2) What is the smallest voltage step size that can be discerned by this ADC?

(3) What is the function of sample-and-hold amplifier?

A3 (1)

$$N_2 = \frac{4.48}{10}(2^8 - 1) \approx 114.24 \approx 114 = (01110010)_2 \tag{4}$$

(2)

$$v_{step} = \frac{10}{2^8 - 1} V \approx 39mv \tag{5}$$

(3) The sample-and-hold circuit can maintain the voltage stability during the AD conversion and reduce the conversion error.

Q4 (10%) A simple Infra-Red Sensor system to detect passing human is presented as in Figure 3. A and B are IR Sensors which will generate different output voltages for different infra-red intensity, and higher voltage level corresponds to high light intensity.

(1) Explain how this system works for counting passing pedestrians.

(2) To increase counting accuracy, usually B is covered with materials that can reflect infra-red light. Explain why.

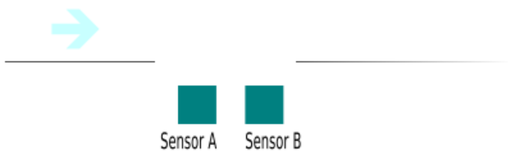

Figure 3: IR-System.

A4 (1) When pedestrians pass over IR Sensor, they will approach and deviate the sensor, which corresponds to voltage pulses V_A at the output of it. We can simply count pulse number for passing pedestrian.

(2) When Sensor B is covered with infra-red reflection materials, it can generate pulses V_B caused by non-infra-red wave. We can reduce wrongly counted number by subtract V_B from V_A to avoid counting noise signal.

Q5 (10%) Exemplify the working principles of sensors that measure: (1) Flow; (2) Temperature; (3) Pressure; (4) Motion; (5) Liquid Level.

A5 Refer to textbook "Principles and Applications of Electrical Engineering" Table 15.1

Q6 (10%) Briefly describe how PID affects motor control.

A6 Refer to lecture 07 slides, page 22-24.

1. **Proportional Gain** K_p: Larger K_p, faster response, but higher instability.
2. **Integral Gain** K_i: Larger K_i, eliminate steady state error, but larger overshoot.
3. **Derivative Gain** K_d: Larger K_d, reduce overshoot, but slower response.

Q7 (15%) Given two Gaussian distributions $N(x_0; \mu_0, \sigma_0)$ and $N(x_1; \mu_1, \sigma_1)$, try to give the expectation and variance of a new distribution which is the product of these two Gaussian distributions.

A7 For detailed proof, refer to the first part of "Products and Convolutions of Gaussian Probability Density Functions"[1].

$$\mu_2 = \mu_0 + \frac{\sigma_0^2 (\mu_1 - \mu_0)}{\sigma_0^2 + \sigma_1^2} \tag{6}$$

$$\sigma_2^2 = \sigma_0^2 - \frac{\sigma_0^4}{\sigma_0^2 + \sigma_1^2} \tag{7}$$

Q8 (15%) Assume the liner estimate system equation is $\mathbf{x}_{t+1} = \mathbf{A}\mathbf{x}_t + \mathbf{w}_t$. Given a second-autoregression random series:

$$x(t) = 1.48x(t-1) - 0.52x(t-2) + \omega_t \tag{8}$$

Kalman Filter is used to estimate $x(t)$ (Here $x(t)$ is a scalar). Try to give the formulations of state transition matrix $\mathbf{A}$ and noise vector $\mathbf{w}_t$.

A8

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -0.52 & 1.48 \end{pmatrix} \tag{9}$$

$$\mathbf{w}_t = \begin{pmatrix} 0 \\ \omega_t \end{pmatrix} \tag{10}$$

[1] The document can be accessed through: `http://www.tina-vision.net/docs/memos/2003-003.pdf`