CENG4480 Homework 1

Due: Oct. 18, 2020

Q1 (10%)

Given the circuit as shown in Figure 1, $R_1 = 2K\Omega$, $R_f = 5K\Omega$, $R_2 = 2K\Omega$, $R_3 = 18K\Omega$, $u_i = 1V$, please compute output voltage u_o.

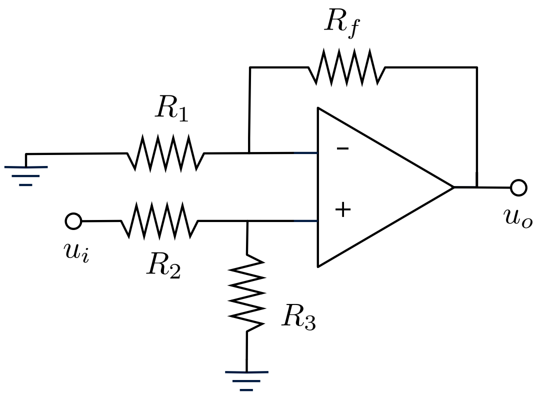

Figure 1: The circuit.

A1 This circuit is a non-inverting circuit.

$$u_- = u_+ = \frac{u_i}{R_2 + R_3} \times R_3 = \frac{1}{2+18} \times 18 = 0.9\ V \tag{1}$$

$$\frac{u_o - u_-}{R_f} = \frac{u_-}{R_1} \tag{2}$$

$$u_o = \frac{u_-}{R_1} R_f + u_- = (1 + \frac{5}{2}) \times 0.9 = 3.15\ V \tag{3}$$

Q2 (10%)

Given a non-inverting amplifier as shown in Figure 2, $R_1 = 4R_2$ and $A_0 = 1000$.

1. Calculate the exact finite gain.
2. Determine the gain difference if the circuit is expected to have an ideal gain under $A_0 = \infty$.

A2 (1): From the proterties of Op Amplifier,

$$V_{out} = A_0(V_{in1} - V_{in2}) \tag{4}$$

Given that,

$$V_{in2} = \frac{R_2}{R_1 + R_2} V_{out} \tag{5}$$

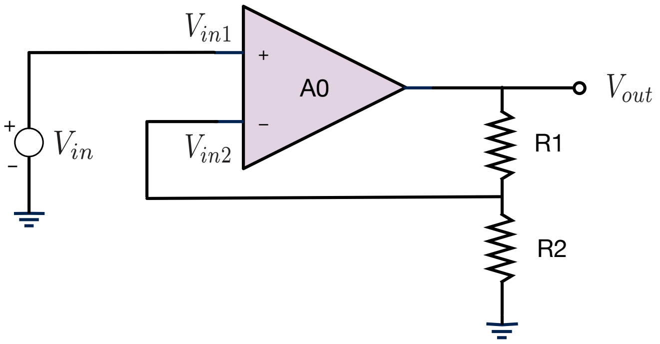

Figure 2: Non-inverting Amplifier.

Substituting into (4) we have,

$$G_{real} = \frac{V_{out}}{V_{in}} = \frac{A_0}{1 + \frac{R_2}{R_1+R_2} A_0} \tag{6}$$

(2):

$$G_{ideal} = (1 + \frac{R_1}{R_2}) \tag{7}$$

Substituting data into Eqs. (6) and (7),

$$G_{real} = 4.98, G_{ideal} = 5 \tag{8}$$

Thus, real circuit gain has a 0.4% difference from ideal gain.

Q3 (10%) Given the inverting amplifier as shown in Figure 3, its supply voltage is $\pm 15V$.

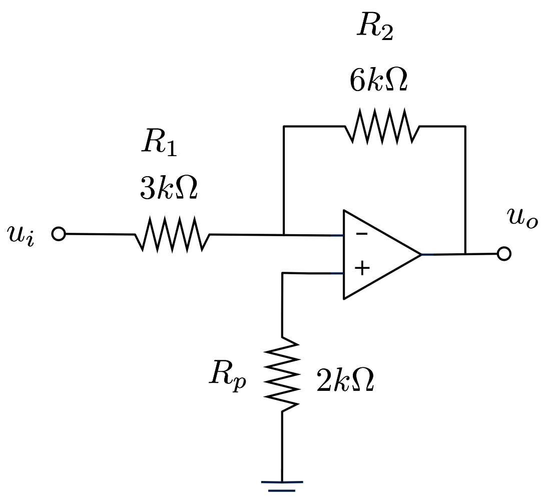

Figure 3: Inverting Amplifier.

1. Compute and sketch transmission curve between u_i and u_o.
2. The input signal is given to be $u_i = 5 sin\omega t(V)$, sketch the waveform of u_o.

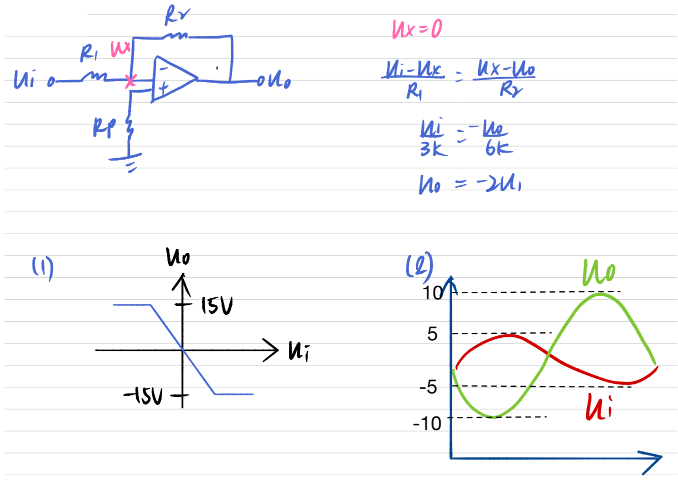

Figure 4: A3 solution

A3 A3 solution is shown in Figure 4:

Q4 (10%)

As shown in Figure 5, $R_f = 2R_1$, $u_i = -2V$, $R_2 = 5K\Omega$, $R_3 = 2K\Omega$, please compute the output voltage u_o.

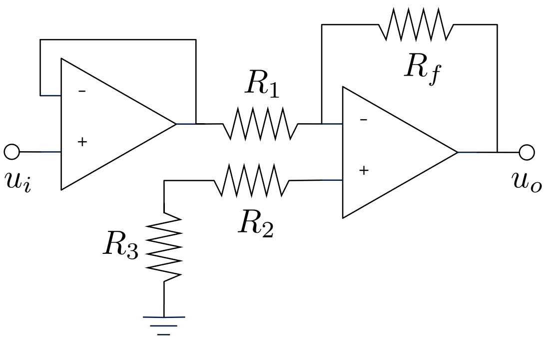

Figure 5: The circuit.

A4 The left stage circuit is a voltage follower, the right stage is an inverting circuit.

$$u_o = -\frac{R_f}{R_1} u_i = 4V \tag{9}$$

Q5 (20%) A differential integrator is shown in Figure 6.

1. Determine the relationship among u_{i1}, u_{i2} and u_o.
2. If we want $u_o = 0V$ when $u_{i2} = 1V$, determine u_{i1}
3. When $t = 0$, $u_{i2} = 1V$, $u_{i1} = 0V$, $u_o = 0V$, determine u_o when $t = 10s$.

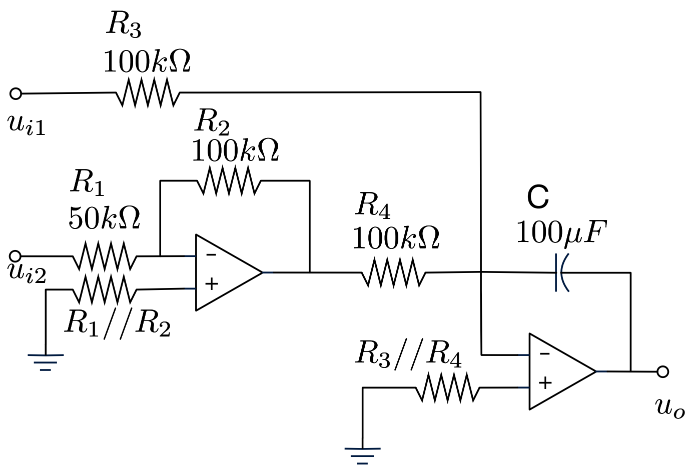

Figure 6: A differential integrator.

A5 Let left op amp output is u_{o2}, according to the inverting amp Gain shown in Lecture 02, we have

$$u_{o2} = -\frac{R_2}{R_1}u_{i2} = -2u_{i2} \tag{10}$$

According to the the relationship between input and output of Integrator shown in Lecture 03, we have

$$u_o(t) = -\frac{1}{C}\int_0^t \frac{u_{i1}(t)}{R_3}dt - \frac{1}{C}\int_0^t \frac{u_{o2}(t)}{R_4}dt \tag{11}$$

$$= \frac{1}{R_3C}\int_0^t (2u_{i2}(t) - u_{i1}(t))dt \tag{12}$$

initial voltage between two ports of capacitor $U_{C(0)} = 0$, we have

$$u_o(t) = \frac{1}{R_3C}\int_0^t (2u_{i2}(t) - u_{i1}(t))dt \tag{13}$$

$$= \frac{1}{100 \times 10^3 \times 100 \times 10^{-6}}\int_0^t (2u_{i2}(t) - u_{i1}(t))dt \tag{14}$$

$$= 0.1\int_0^t (2u_{i2}(t) - u_{i1}(t))dt \tag{15}$$

$$u_o = 0.1\int_0^t (2u_{i2}(t) - u_{i1}(t))dt = 0 \tag{16}$$

$$u_{i1}(t) = 2u_{i2}(t) = 2V \tag{17}$$

$$u_o(t) = 0.1 \int_0^t (2u_{i2}(t) - u_{i1}(t))dt \quad (18)$$
$$= 0.1 \int_0^t (2 \times 1)dt \quad (19)$$
$$= 0.2t \quad (20)$$
$$= 0.2 \times 10 \quad (21)$$
$$= 2V \quad (22)$$

Q6 (10%) Given a low-pass filter as shown in Figure 7.

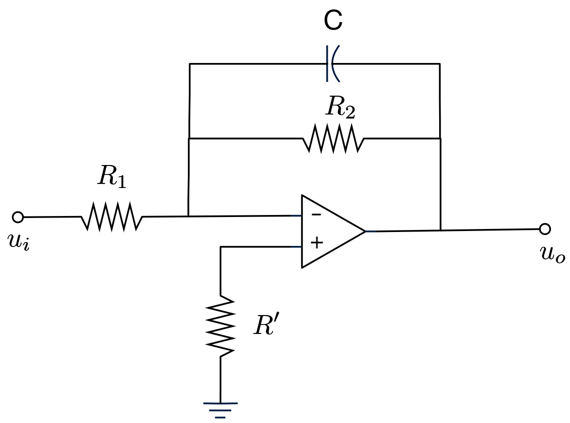

Figure 7: A low-pass filter.

1. If $R_1 = 10K\Omega$, $R_2 = 200K\Omega$, determine low-frequency gain $A_u(dB)$;
2. If cutoff frequency $f_c = 6Hz$, determine C value.

A6

$$u_o(j\omega) = -\frac{R_2 // \frac{1}{j\omega C}}{R_1} u_i(j\omega) \quad (23)$$

So

$$A_u(j\omega) = \frac{u_o(j\omega)}{u_i(j\omega)} = -\frac{R_2}{R_1} \frac{1}{1 + j\omega R_2 C} = \frac{A_{u_o}}{1 + j\frac{\omega}{\omega_c}} \quad (24)$$

where $A_{u_o} = -\frac{R_2}{R_1}$ is low-frequency gain, $\omega_c = \frac{1}{R_2 C}$ is cutoff angular frequency. If $R_1 = 10K\Omega$, $R_2 = 200K\Omega$, low-frequency gain

$$20 \log \frac{R_2}{R_1} = 20 \log \frac{200}{10} = 26.02dB \quad (25)$$

cutoff frequency:

$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi R_2 C} = \frac{1}{2\pi \times 200 \times 10^3 \times C} = 6Hz \quad (26)$$

so

$$C = \frac{1}{2\pi \times 200 \times 10^3 \times f_c} = \frac{1}{2\pi \times 200 \times 10^3 \times 6} = 1.33 \times 10^{-7}F = 0.133\mu F \quad (27)$$

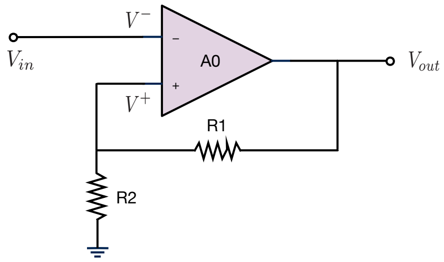

Figure 8: Schmitt Trigger.

Q7 (20%) Let us consider the Schmitt Trigger shown in Figure 8

1. Due to the manufacturing defects, a parasitic resister R_3 occurs between the output node and ground, calculate the reference voltages.
2. If the parasitic device is a capacitor C, sketch v_{out} versus v_{in}. Label the key coordinates on the curve.

A7

1. According to the properties of comparator, when v_{in} is small, $v_{out} = v_{sat}$ and

$$\frac{v^+}{R_2} = \frac{v_{out} - v^+}{R_1}, \tag{28}$$

i.e.,

$$v^+ = \frac{R_2}{R_1 + R_2} v_{sat}. \tag{29}$$

Similarly, if v_{in} is large, we have

$$v^+ = -\frac{R_2}{R_1 + R_2} v_{sat}. \tag{30}$$

Therefore two reference voltages are given by $\frac{R_2}{R_1+R_2} v_{sat}$ and $-\frac{R_2}{R_1+R_2} v_{sat}$.

2. v_{out} start to change when v_{in} reaches references above. However, due to the existing capacitor, voltage cannot change immediately (Changes fast, then slowly), as shown in Figure 9.

Q8 (10%) An ADC is used to sample an analog signal.

1. If the maximum frequency of the analog signal is $10kHz$, determine the minimum sampling frequency.
2. As shown in Figure 10, if the ADC is integrating ADC with 15 bits and clock frequency is $2MHz$, determine the maximum conversion frequency.

A8 $20kHz$

conversion time $T = 2^{n+1}T_c$, $T_c = 2/10^6$, $T = 32.768ms$.

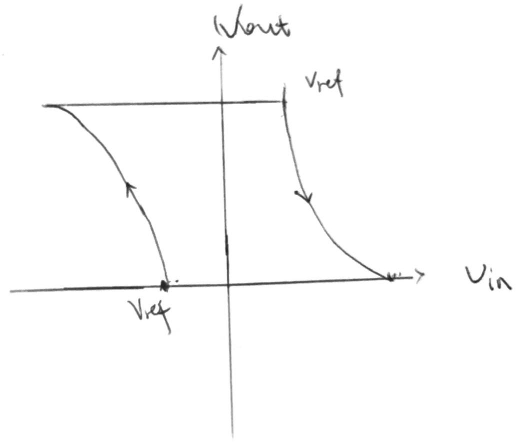

Figure 9: A6(2).

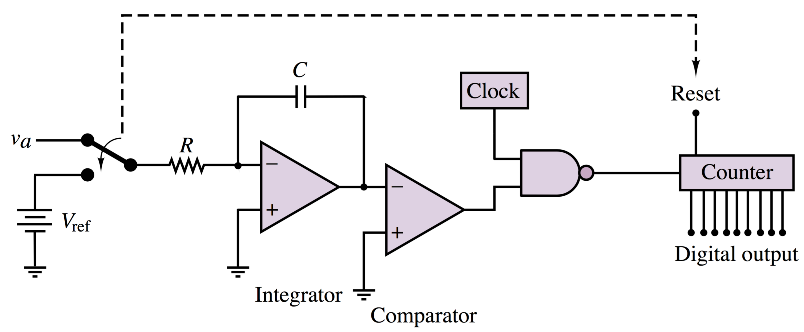

Figure 10: Integrating ADC.