CENG 4480 Midterm (Fall 2017)

Name: ___________
ID: ___________

Solutions

Q1 **(40%)** Check or fill the correct answer:

1. A circuit where the input signal power is less than the output signal power is called **amplifier/attenuator**.
2. In an ideal op amplifier, $V_{in+} > / = / < V_{in-}$, since it has **infinite/finite** open-loop gain.
3. A amplifier with input voltage of 1mv and output voltage 1V has gain ___ dB.
4. A capacitor can be regarded as an open circuit when a **high/low** frequency signal is input.
5. A Schmitt Trigger based on inverting comparator has a **positive/negative** feedback.
6. ___ is the minimum number of bits required to digitize an analog signal with a resolution of 1%. (Resolution is the ratio between minimum voltage that can be sensed and the input voltage range.)
7. **high-pass filter/sample-and-hold circuit** is used to reduce the glitch.
8. **Accelerometer/Gyroscope/Strain Gauge** is usually used to measure rotation angle.
9. Light-to-voltage optical sensors contains **photodiode/amplifier** to sense light intensity change.
10. In PID control, decreasing proportional gain will lead to the **faster/slower** response. And we will get **faster/slower** elimination of steady state error by adding integral gain, while **increasing/decreasing** settling time and overshot with a larger derivative gain.

Q2 **(20%)** The integrator of Fig. 1 senses an input signal given by $V_{in} = A \cos \omega t$. Determine the output signal amplitude if $A_0 = \infty$.

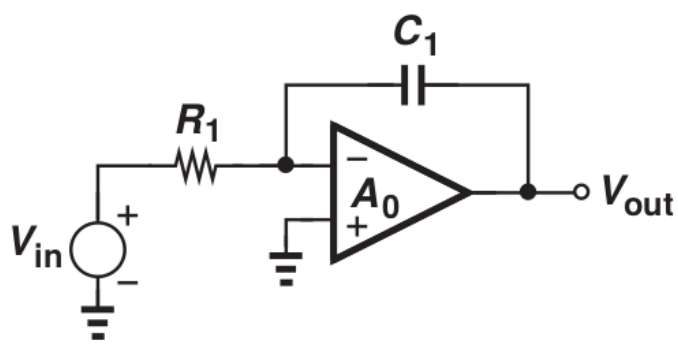

Figure 1: Figure of Q2

Q3 **(20%)** Try to use discrete incremental PID formulations to calculate $\Delta u(t)$. Some notations and values of parameters are given:

- $u(t)$ is the output of a controller in the tth measurement interval.

- $e(t)$ is the error between the target value and measurement value in the tth measurement interval. The error is measured every T time interval ($T = 0.001$).
  And $e(t) = 2$, $e(t-2) = 5$ and $e(t-1) = 3$.

- The numerical values of PID parameters, K_p, K_i and K_d, are 1, 50, 0.001 respectively.
  (Hint: The formulation of continuous PID is $u(t) = K_p e(t) + K_i \int_0^t e(t)\,\mathrm{d}t + K_d \frac{\mathrm{d}e(t)}{\mathrm{d}t}$)

Q4 (20%) The general equation of a liner estimate system is like $\mathbf{x}_{t+1} = \mathbf{A}\mathbf{x}_t + \mathbf{w}_{t+1}$. Given a second-autoregression random series:

$$x(t) = 2.32x(t-1) - 0.76x(t-2) + \omega_t \tag{1}$$

Kalman Filter is used to estimate $x(t)$ (Here $x(t)$ is a scalar). Try to give the formulations of state transition matrix $\mathbf{A}$ and noise vector $\mathbf{w}_t$.

A1
1. amplifier
2. =, infinite
3. 60
4. low
5. positive
6. 7
7. sample-and-hold circuit
8. Gyroscope
9. photodiode
10. slower, faster, decreasing

A2 It is easy to know,

$$V_{out} = -\frac{1}{R_1C_1}\int V_{in}dt \tag{2}$$

i.e.,

$$V_{out} = -\frac{A}{R_1C_1\omega}\sin\omega t \tag{3}$$

Output signal amplitude is $\frac{A}{R_1C_1\omega}$

A3

$$u(t) = K_p * e(t) + K_i * \sum e(t) * T + K_d * \frac{e(t) - e(t-1)}{T} \tag{4}$$

$$u(t-1) = K_p * e(t-1) + K_i * \sum e(t-1) * T + K_d * \frac{e(t-1) - e(t-2)}{T} \tag{5}$$

$$\Delta u(t) = K_p * (e(t) - e(t-1)) + K_i * e(t) * T + K_d * \frac{e(t) - 2e(t-1) + e(t-2)}{T} \tag{6}$$

So $\Delta u(t) = 0.1$

A4 The random series is extended as:

$$\begin{cases} x(t-1) & = & 0 \cdot x(t-2) & + & 1 \cdot x(x-1) & + & 0 \\ x(t) & = & -0.76 \cdot x(t-2) & + & 2.32 \cdot x(t-1) & + & \omega_t \end{cases} \tag{7}$$

Its matrix form is

$$\begin{bmatrix} x(t-1) \\ x(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.76 & 2.32 \end{bmatrix} \cdot \begin{bmatrix} x(t-2) \\ x(t-1) \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_t \end{bmatrix} \tag{8}$$

Let $\boldsymbol{\chi}(t) = \begin{bmatrix} x(t-1) \\ x(t) \end{bmatrix}$, $\boldsymbol{\chi}(t-1) = \begin{bmatrix} x(t-2) \\ x(t-1) \end{bmatrix}$, $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -0.76 & 2.32 \end{bmatrix}$ and $\mathbf{w}_t = \begin{bmatrix} 0 \\ \omega_t \end{bmatrix}$, Equation (8) is equivalent to $\boldsymbol{\chi}(t) = \mathbf{A} \cdot \boldsymbol{\chi}(t-1) + \mathbf{w}_t$.