Stochastic Differential Equation Approach to Model BitTorrent-like P2P Systems

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***Abstract*— In this paper, we propose to model the dynamics of BitTorrent (BT) P2P file sharing systems using the stochastic differential equation method. Unlike previous approach, our method can capture more realistic network environment and peers behavior. Closed-form solutions of various performance measures such as the average number of downloaders, seeders, the system throughput and file downloading time are derived. We also validate our mathematical results via simulation and show that not only our mathematical model can closely track the dynamics of BT-like systems, but the model has a much higher accuracy than previous proposed methods. Also, many important properties can be derived from the close-form solution such as performance scalability, sensitivity of the measurements to various system parameters. We believe the proposed method can provide better understanding in the design and analysis of BT-like P2P systems.**

I. Introduction

Recently, peer-to-peer (P2P) file sharing architectures for tens of thousand of end hosts are generating an increasing amount of traffic on today's Internet. This form of paradigm is reshaping the way new network applications are being designed. For example, one can easily find P2P softwares for multimedia file sharing (i.e., video and audio files), live video streaming applications [1].

Among various P2P applications, the most popular application is still for file sharing. Compared to the traditional client/server paradigm, the P2P paradigm has much better scalability. When the number of users increases, the performance (i.e., file downloading time) for the client/server architecture will degrade substantially. On the other hand, P2P architecture has an inherent property that more users will actually improve the file downloading performance. This property is especially true for the latest generation of P2P softwares that follow the BitTorrent (BT) protocol [2]. The major advantage of the BitTorrent protocol is that when there are multiple downloads of the same file, the downloaders also need to upload a part of the file to other downloaders, and thereby making it possible to support a large number of downloaders and reduce the time for file downloading.

In this paper, the main contributions of our work are:

- We develop a fluid model for BT-like P2P systems based on the "*stochastic differential equation*" technique [3] which allows us to obtain and analyze the expected transient and the steady state performance measures.
- The analytical model is validated by a discrete event simulation which is detailed enough to capture many of BT's features[1]. The analytical results and measurements provide us the insights for designing a BT-like protocol. Also, as compared to the simple fluid model in [4], our model focuses more on characterizing details of heterogeneous peers with reasonable network topology and network parameters, and at the same time, maintains the model simplicity and mathematical tractability.

The balance of our paper is as follows. In Section 2, we provide a basic introduction to BitTorrent and a brief review of related work. In Section 3, we present the mathematical model to describe the dynamics of a BT-like P2P system as well as its performance measures. Performance evaluations and model validation are presented in Section 4. Section 5 concludes.

II. Previous Work

Recently, there are a number of analytical and measurement-based studies of BT-like systems. In [5], authors present a measurement results collected during a five-month period that involves thousands of peers. The authors assessed the performance of the algorithms and mechanisms used by BT. In [6], authors present an eight-month trace based study and measurement results of the popularity and availability. In [7], the authors analyze the measurement result collected by the modified client in a BT network and propose a P2P-based streaming protocol. In [8], authors study the ability of the BT protocol to disseminated very large files among peers and present measurement results over a duration of four months.

There are also simulation based studies on BT systems. In [9] the authors suggest that network coding scheme can be used to improve the performance of BT-like systems. In [10], authors conduct various simulation-based experiments to investigate the effect of network parameters and system settings on the performance of file downloading.

At the analytical end, authors in [11], [12] propose a coarse-grain Markovian model to represent a P2P file sharing system. However, this Markovian model cannot capture many

[1]Some of the previous research results did not perform model validation.

important properties of a BT-like system. Furthermore, these is no closed form solutions to study the steady state. One can only use numerical method for some performance measures. To overcome the computation problem in [11], [12], authors in [4] propose a fluid model and a set of differential equations to describe the dynamics of BT-liked systems and discuss issues like incentive mechanisms and free-riding. However, the model also fails to capture many intrinsic and important properties of BT-like P2P systems such as node degree and number of file sharing connections. Also, these previous works do not consider the underlying overlay topology and treat the effective throughput of peers as a constant. In [13], authors develop a detailed Markovian model to investigate the scalability and effectiveness of a P2P system. However, the result is more of theoretical interest only since the model has a huge state space and it is difficult to analyze. Instead, one has to resort to asymptotic analysis. In [14], authors extend the model in [4] to illustrate the performance issue of providing service differentiation in a BT-like system. Nevertheless, similar to [4] wherein many simplified assumptions are made and essential network parameters are omitted which impede fundamental understanding on BT systems.

III. Mathematical Model for System Dynamics

In this section, to represent the dynamics and evolution of a BitTorrent-like P2P system, we present a fluid model with simplified state space using the stochastic differential equation approach [3]. Various performance measures, such as the average number of leechers, the average number of seeders, the average file downloading time and the overall system throughput, are derived.

Consider a BitTorrent-like P2P system that distributes a given file $\mathcal{F}$ to a large number of cooperative peers. The file is divided into M orthogonal chunks such that $\mathcal{F} = \mathcal{F}_1 \cup \mathcal{F}_2 \cup \cdots \cup \mathcal{F}_M$, $\mathcal{F}_i \cap \mathcal{F}_j = \emptyset$ for $i \neq j$ and $\mathcal{F}_i$ is the i^{th} chunk of the file. For simplicity of analysis, we assume no network coding or erasure code is applied in the file sharing process. Typically, the number of chunks M is in the order of several hundreds. Based on BT's definition, a *seeder* has all M chunks of $\mathcal{F}$ while a *leecher* has a subset of $\mathcal{F}$. Assume at time t, there are $N(t)$ peers in the system to obtain and share the file $\mathcal{F}$, and new peers arrive according to a Poisson arrival process with rate λ. By the help of a tracker, each peer maintains a connection with a probability of $\rho \leq 1$ to other peers as its neighbors. So every node in the overlay topology has a degree of $\rho(N(t)-1) \approx \rho N(t)$. The tracker can increase this degree by sending more peers to the new peer to increase ρ. For each connection, the average downloading rate is μ. Each peer is constrained by a maximum transfer rate B including the downloading and the uploading rates[2]. Although a peer can keep logical connections to many peers, one peer can have at most B/μ uploading and/or downloading connections simultaneously. After collecting all chunks of $\mathcal{F}$, a leecher becomes a seeder and may serve others by uploading chunks. A seeder can choose to leave a BT-like system and the average departure rate is γ (i.e., $1/\gamma$ is the average time a seed stays in the BT system). We also let c_i represent the number of chunks that peer i is holding. Table I summarizes the notations we use in the mathematical model.

[2]As reported in [15], most popular BitTorrent clients provide this kind of feature

M	the number of chunks of the given file $\mathcal{F}$.
s	the size of a chunk (in $bits$).
$N(t)$	r.v.(random variable) denoting number of peers at time t.
λ	the average arrival rate of new leechers.
γ	the departure rate of the seeders.
μ	the average downloading rate (in bps) between two peers.
B	the maximize transfer bandwidth (in bps) of a peer.
ρ	probability that two peers are connected.
c_i	r.v. denoting the number of chunks $peer_i$ holds.
$X_1(t)$	r.v. denoting the number of type-1 peers at time t.
$X_2(t)$	r.v. denoting the number of type-2 peers at time t.
$Y(t)$	r.v. denoting the number of seeders at time t.

TABLE I

NOTATION FOR MATHEMATICAL MODELS

Assume the chunks are uniformly distributed among peers, which actually could be ensured by the *rarest-first* policy. Let peer i and peer j have c_i and c_j chunks respectively, where $c_i, c_j \in \{0, 1, \ldots, M\}$. Let us derive the probability that peer i can obtain *at least* one useful chunk from peer j, which we denote as $P_{i,j}$. When $c_i < c_j$, it is clear that $P_{i,j} = 1$. When $c_i \geq c_j$, we have:

$$\begin{aligned} P_{i,j} &= 1 - \text{P[peer } j \text{ cannot provide any useful chunk to peer } i] \\ &= 1 - \text{P[chunks of peer } j \text{ are subset of chunks of peer } i] \\ &= 1 - \frac{\binom{c_i}{c_j}}{\binom{M}{c_j}} = 1 - \frac{c_i \cdot (c_i - 1) \cdots (c_i - c_j + 1)}{M \cdot (M-1) \cdots (M - c_j + 1)}. \end{aligned} \quad (1)$$

We distinguish three types of peers: Type 1 peer is a leecher that holds a few chunks (i.e., say less than half of the M chunks), while type 2 peer is a leecher that holds most but not all chunks. Type 3 peer is to represent a seeder in the system. The probability $P_{i,j}$ in (1) can be simplified based on the following cases:

- **Case 1:** If peer i is of type 1 or type 2, and peer j is of type 3, then clearly $P_{i,j} = 1$ since a seeder can always help a leecher.
- **Case 2:** If peer i is of type 1 and peer j is of type 1 or type 2, then c_i/M is very small and we have $P_{i,j} \geq 1 - (c_i/M)^{c_j} \approx 1$.
- **Case 3:** If peer i is of type 2 and peer j is of type 1, then c_i/M is close to 1 but since c_j is small, we have $P_{i,j} \approx 0$.
- **Case 4:** If peer i and peer j are of type 2, then c_j is large and $(c_i/M)^{c_j} \approx 0$, so $P_{i,j} \approx 1$.

To represent the *heterogeneity* of peers while keeping the model simple and analytically tractable, we assign $P_{i,j}$ to have two possible values: 0 or 1 according to the types of peer i and j.

Let $X_1(t)$, $X_2(t)$ and $Y(t)$ be the random variables representing the number of type-1 peers, type-2 peers and seeders in the system at time t. By case 1 and 2 of the analysis of (1), type-1, type-2 peers and seeders could help type-1 peers in file download. Also, type-2 peers and seeders could help type-2 peers based on case 1, 2 and 4. Let $D_i(t)$, $U_i(t)$ denote the random variables of the downloading and uploading rates for $peer_i$ at time t. When there is no bandwidth constraint (i.e., B is infinitely large), we have:

$$E[D_i(t)] = \begin{cases} \mu\rho\,(E[X_1(t)]+E[X_2(t)]+E[Y(t)]) & i \text{ is type-1} \\ \mu\rho\,(E[X_2(t)] + E[Y(t)]) & i \text{ is type-2.} \end{cases} \quad (2)$$

When the bandwidth constraint B is considered, it means that for each peer i, $D_i(t)+U_i(t) \leq B$ needs to be satisfied. From the system's perspective, we have the conservation rules:

$$\sum_{j=1}^{N(t)} D_j(t) + \sum_{j=1}^{N(t)} U_j(t) \leq BN(t), \quad (3)$$

$$\sum_{j=1}^{N(t)} D_j(t) = \sum_{j=1}^{N(t)} U_j(t). \quad (4)$$

Substitute (4) to (3) and taking the expectation, by the Wald's Equation [16] we have:

$$E[D_i(t)] \leq B/2. \quad (5)$$

Combining (2) and (5) and let $D^{(1)}(t)$ and $D^{(2)}(t)$ be the random variables denoting the downloading rate at time t for type-1 and type-2 peer respective, we have:

$$\begin{aligned} &E[D^{(1)}(t)] \approx \min\{\mu\rho(E[X_1(t)] + E[X_2(t)] + E[Y(t)]), B/2\} \\ &E[D^{(2)}(t)] \approx \min\{\mu\rho(E[X_2(t)] + E[Y(t)]), B/2\} \end{aligned} \quad (6)$$

We can now present the mathematical model that captures the dynamics of the P2P system. The model is based on the stochastic differential equation [3]. First, the arrival process of new peers is modeled as a Poisson counter process $N(t)$ with an average arrival rate λ. The Poisson counter has the following properties:

$$dN(t) = \begin{cases} 1 & \text{at Poisson arrival} \\ 0 & \text{elsewhere} \end{cases} \quad (7)$$

$$E[dN(t)] = \lambda dt \quad (8)$$

Let $X_1(t)$ and $X_2(t)$ denote the number of type-1 and type-2 leechers at time t while $Y(t)$ denote the number of seeders in the system at time t. The following describes the rate of change of these three important variables:

$$\begin{aligned} dX_1(t) &= dN(t) - \tfrac{D^{(1)}(t)X_1(t)dt}{sM/2}, \\ dX_2(t) &= \tfrac{D^{(1)}(t)X_1(t)dt}{sM/2} - \tfrac{D^{(2)}(t)X_2(t)dt}{sM/2}, \\ dY(t) &= \tfrac{D^{(2)}(t)X_2(t)dt}{sM/2} - \gamma Y(t)dt. \end{aligned} \quad (9)$$

The rate of change of $X_1(t)$ is affected by the number of new arrival, which is denoted as $dN(t)$, and the number of peers that transfer from type-1 to type-2 which is denoted by $\frac{D^{(1)}(t)X_1(t)dt}{sM/2}$, where $sM/2$ represents the size of a half of the file $\mathcal{F}$, and $D^{(1)}X_1(t)dt$ represents the amount of new information that all $X_1(t)$ type-1 peers collect in dt. Similarly, the transfer rate from type-2 peers to seeders is $\frac{D^{(2)}(t)X_2(t)dt}{sM/2}$. Lastly, the departure rate of a seeder is γ, so the total departure rate of all seeders is represented by $\gamma Y(t)$ After taking the expectation of (9), we have:

$$\begin{aligned} dE[X_1(t)] &\approx E[dN(t)] - \tfrac{E[D^{(1)}(t)]E[X_1(t)]dt}{sM/2}, \\ dE[X_2(t)] &\approx \tfrac{E[D^{(1)}(t)]E[X_1(t)]dt}{sM/2} - \tfrac{E[D^{(2)}(t)]E[X_2(t)]dt}{sM/2}, \\ dE[Y(t)] &\approx \tfrac{E[D^{(2)}(t)]E[X_2(t)]dt}{sM/2} - \gamma E[Y(t)]dt. \end{aligned} \quad (10)$$

Note that the above equations are approximations because we are assuming the independence of $D^i(t)$ and $X_i(t)$.

To study the *steady-state* performance, we let $dE[X_1(t)] = dE[X_2(t)] = dE[Y(t)] = 0$. To simplify notation further, we use $\bar{V}$ to represent the expected value of the random variable V and we let $\alpha = \frac{2\mu\rho}{sM}$ and $\beta = \frac{B}{2\mu\rho}$. Equation (10) can now be re-written as:

$$\begin{aligned} &0 = \lambda - \alpha\bar{X}_1 \min\{\bar{X}_1 + \bar{X}_2 + \bar{Y}, \beta\} \\ &0 = \alpha\bar{X}_1 \min\{\bar{X}_1 + \bar{X}_2 + \bar{Y}, \beta\} - \alpha\bar{X}_2 \min\{\bar{X}_2 + \bar{Y}, \beta\} \\ &0 = \alpha\bar{X}_2 \min\{\bar{X}_2 + \bar{Y}, \beta\} - \gamma\bar{Y} \end{aligned} \quad (11)$$

To solve these steady state equations, we classify the (11) into three cases:

Case 1 $\bar{X}_1 + \bar{X}_2 + \bar{Y} < \beta$ and
Case 2 $\bar{X}_2 + \bar{Y} < \beta < \bar{X}_1 + \bar{X}_2 + \bar{Y}$ and
Case 3 $\beta < \bar{X}_2 + \bar{Y}$.

The first case implies that the uploading and downloading are *not* constrained by the bandwidth B. This occurs when the peers have broadband access to the Internet, or when the peer's arrival rate is low so there are only few peers in the system. For the second case, type-1 peers are constrained by bandwidth B while type-2 peers are not constrained by this bandwidth limit. The reason is there are more peers who can help type-1 peers than type-2 peers. Hence it is possible for the former to be saturated by bandwidth, yet not the latter. For the last case, all peers are constrained by the bandwidth B in the file sharing process. This case occurs when peers have a low bandwidth connection to the Internet or the file is very popular so that the peer's arrival rate is very high and there are many peers in the system. We can solve $\bar{X}_1$,$\bar{X}_2$,$\bar{Y}$ respectively in these three cases (please refer to [17] for derivations).

Theorem *1: The average number of type 1 leechers ($\bar{X}_1$), type 2 leechers ($\bar{X}_2$) and seeders ($\bar{Y}$) can be expressed as:*

$$\begin{aligned} \bar{X}_1 &= \begin{cases} \frac{\sqrt{5}-1}{2}\sqrt{\frac{sM\lambda}{2\mu\rho}} - \frac{\lambda}{4\gamma} & \text{Case 1,} \\ sM\lambda/B & \text{Case 2 and 3} \end{cases} \\ \bar{X}_2 &= \begin{cases} \sqrt{\frac{sM\lambda}{2\mu\rho}} - \frac{\lambda}{2\gamma} & \text{Case 1 and 2,} \\ sM\lambda/B & \text{Case 3} \end{cases} \\ \bar{Y} &= \lambda/\gamma \qquad \text{Case 1,2, and 3.} \end{aligned} \quad (12)$$

Proof: Please refer to [17] for derivation. ∎

Theorem *2: Let $\bar{T}_d$ denote the average downloading time for a file, which is the average time it takes for a peer to obtain all M unique chunks of $\mathcal{F}$. We have the following results:*

$$\bar{T}_d = \begin{cases} \frac{1+\sqrt{5}}{2}\sqrt{\frac{sM}{2\mu\rho\lambda}} - \frac{3}{4\gamma} & \text{Case 1,} \\ \sqrt{\frac{sM}{2\mu\rho\lambda}} + \frac{sM}{B} - \frac{1}{2\gamma} & \text{Case 2,} \\ \frac{2sM}{B} & \text{Case 3.} \end{cases} \quad (13)$$

Proof: By Little's Law [18], $\bar{T}_d$ is given by $\bar{T}_d = \frac{\bar{X}_1 + \bar{X}_2}{\lambda}$. Based on Theorem 1, the above results can be easily derived. ∎

Theorem *3: Let $\bar{T}_p$ denote the average system throughput of the BT-like P2P system, $\bar{N} = \bar{X}_1 + \bar{X}_2 + \bar{Y}$. We have the following result:*

$$\bar{T}_p = \begin{cases} O(\bar{N}^2) & \text{Case 1,} \\ O(\bar{N}) & \text{Case 2 and 3} \end{cases} \quad (14)$$

Proof: The proof is in [17]. ∎

Remark 1: Scalability of BitTorrent-like P2P networks: Based on the total throughput of the system in steady state given by (14), one can find that the P2P system scales well with the number of peers. Case 1 represents the system under a low arrival rate, therefore a small number of peers exist in the system. The throughput of the system is of the order of $O(\bar{N}^2)$. When there are more peers in the systems (i.e., in case 2 and 3), the system throughput is linearly proportional to the number of peers. So the system performance will not degrade with an increased number of peers in the system.

Remark 2: Sensitivity of downloading time to arrival rate: The intensity of the arrival rate represents the popularity of the file. To understand the impact of file popularity to the performance of BT-like P2P systems, we consider the rate of change of $\bar{T}_d$ when one increases the peer's arrival rate λ. Based on the expression of $\bar{T}_d$, we have:

$$\frac{\partial \bar{T}_d}{\partial \lambda} = \begin{cases} -\frac{1+\sqrt{5}}{4\sqrt{\alpha}}\lambda^{-3/2} & \text{Case 1,} \\ -\frac{1}{2\sqrt{\alpha}}\lambda^{-3/2} & \text{Case 2,} \\ 0 & \text{Case 3.} \end{cases}$$

For case 1 and 2 of our model, the average downloading time decreases when the arrival rate λ increases; in case 3, the rate of change of $\bar{T}_d$ is not related to λ. This means the more popular a file is (larger λ), the smaller the average downloading time. Therefore the BT-like system scales well with the popularity.

Remark 3: Effect of Seeders: Since γ represents the departure rate for seeders, $T_s = 1/\gamma$ is the average time a seeder stays in a P2P system. For case 1 and 2, when T_s increases, there will be more seeders in the system to provide the uploading service, therefore, the average downloading time $\bar{T}_d$ will decrease. Notice that:

$$\frac{\partial \bar{T}_d}{\partial T_s} = \begin{cases} -3/4 & \text{Case 1,} \\ -1/2 & \text{Case 2,} \\ 0 & \text{Case 3} \end{cases}$$

Having more seeders will reduce the loading time. But when all peers are saturated due to the bandwidth limit, having more seeders will not improve the performance of end users.

Consider an extreme case of $T_s = 0$, that is, a peer will leave the system immediately after it downloads the entire file $\mathcal{F}$.

$$\lim_{\gamma\to\infty} \bar{T}_d = \begin{cases} \frac{1+\sqrt{5}}{2}\sqrt{\frac{sM}{2\mu\rho\lambda}} & \text{Case 1,} \\ \sqrt{\frac{sM}{2\mu\rho\lambda}} + \frac{sM}{B} & \text{Case 2,} \\ \frac{2sM}{B} & \text{Case 3.} \end{cases}$$

The above expression implies that peers can still obtain the file, though with higher downloading time, without the help of many seeders.

Remark 4: Effect of the connection probability ρ: A close examination of (13) reveals that T_d is a function of the connectivity parameter ρ for case 1 and 2 but not case 3. Increasing the value of ρ will reduce the value of $\bar{T}_d$. This is due to the fact that a peer has more connections to reduce its downloading time, as long as it is not saturated by its own bandwidth limit. In case 1 and case 2, increasing ρ will decrease $\bar{T}_d$, because lager ρ might bring more downloadings for peers. In case 3, ρ will not affect T_d because the system is operating in the saturated mode. One may think a larger value of ρ will always benefit a peer. However it is important to note that larger value of ρ will also cause peers to keep too many TCP connections to other peers. Hence a large value of ρ will burden the peers with too many connection overheads and eventually leads to saturating peers' bandwidth. Since ρ is affected by the number of peers returned to the new peer from the tracker (now 30-60 by default), a proper selection of this number is an interesting and practical problem. For example in case 3, because $\bar{T}_d$ is only related to the bandwidth B and the file size, the system only needs to choose the proper value of ρ which keeps the system stays in case 3.

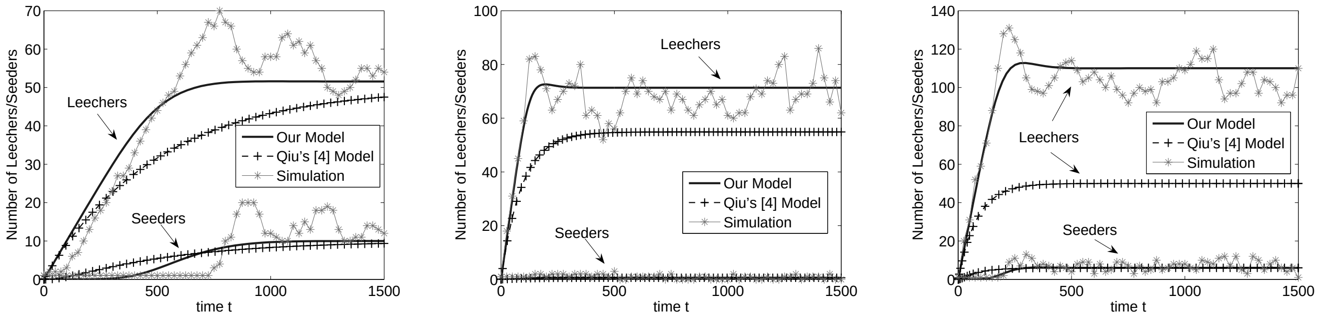
Remark 5: Effect of bandwidth B: Considering the marginal utilization of B:

$$\frac{\partial \bar{T}_d}{\partial B} = \begin{cases} 0 & \text{Case 1,} \\ -\frac{sM}{B^2} & \text{Case 2,} \\ -\frac{2sM}{B^2} & \text{Case 3} \end{cases}$$

For case 1, the bandwidth is not fully utilized so $\bar{T}_d$ is not affected by B, and more bandwidth is not helpful at all in this case. But the marginal utilization of B in case 3 is two times as that in case 2. So by dedicating more bandwidth, a peer could get better performance in these two cases. Given the above analysis, one can better anticipate the need in a BT-like file sharing system.

IV. Performance Evaluation and Validation

To validate our analytical results, we implement a discrete event simulator for a BitTorrent-like file sharing system. The input of the simulator are parameters such as arrival rate, transfer rate between peers, departure rate of seeds, connection



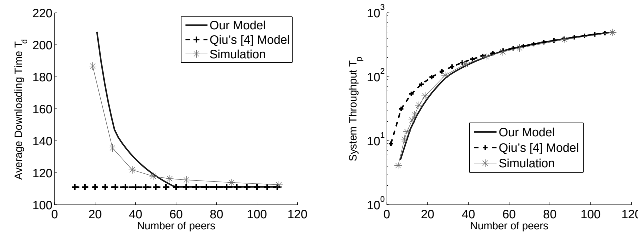
(a) (b) (c)

Fig. 1. Dynamics of peer evolutions for three different cases

probability, transmission bandwidth of peers, etc. Our simulator models the behaviors of peers such as joining the system, making connections to neighboring nodes, selecting chunks for download, transfer chunks, updating the chunk bitmaps, seeding and also departures of seeders.

To simplify the simulation complexity, the simulator does not capture the following aspects: 1) The tit-for-tat incentive mechanism. Note that in some BT-implementations, the standard "choking" mechanism is rarely used. Instead, they allow users to set the maximum bandwidth allocated to the file sharing process. 2) The cross-traffic on shared overlay is not considered. So in our simulation, all traffics are end-to-end between any two peers in a BT-system. 3) The network propagation delay is ignored since the chunk size (or the chunk's transmission time) is relatively large compared to the propagation delay.

Exp. 1) Accuracy in estimating number of peers: In the following experiments, we consider the accuracy of the proposed mathematical model in estimating $E[X_1(t)]$, $E[X_2(t)]$ and $E[Y(t)]$. In Fig.1, we compare the average number of leechers ($E[X_1(t)] + E[X_2(t)]$) and the average number of seeders ($E[Y(t)]$) with the simulation results. Fig.1(a) illustrates the case that the peer's arrival rate is $\lambda = 0.1$, seeder's departure rate is $\gamma = 0.01$, the transfer rate is $\mu = 0.1$ between two peers, the maximum transfer bandwidth of a peer is $B = 2$ and the connection probability is $\rho = 0.25$. The setting represents the situation that peers with poor download bandwidth, the maximum transfer rate between peers is low. Because the peer's arrival rate is low, so the file is not that popular. One can see that our model can accurately track the dynamics of the leechers and seeders, while model based on [4] is only accurate in estimating the number of leechers and seeders in the steady state case. Fig.1(b) illustrates the case that the peer's arrival rate is $\lambda = 0.6$, seeder's departure rate $\gamma = 1.0$, peer's downloading bandwidth is $\mu = 0.3$, peer's maximum transfer bandwidth is $B = 12$ and the connection probability is $\rho = 0.25$. In this setting, the file is more popular so the peer's arrival rate is higher. Also, the peers have better downloading rate and a higher maximum transfer bandwidth. However, the seeder's departure rate is also higher than the previous experiment. Again, our model can accurately track the dynamics of the leechers and seeders, while model based on [4] underestimates the number of leechers in the system. Lastly, Fig.1(c) illustrates the case that the peer's arrival rate is $\lambda = 0.6$, seeder's departure rate $\gamma = 0.1$, downloading bandwidth between peers is $\mu = 0.3$, peer's maximum transfer bandwidth is $B = 12$ and the connection probability is $\rho = 0.1$. Note that our model can accurately track the dynamics of the leechers and seeders, while model based on [4] significantly underestimates the number of leechers in the system.



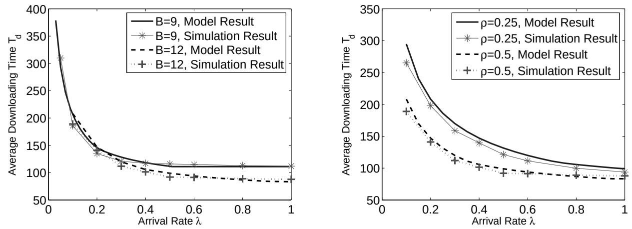
(a) $\bar{T}_d$ as the function of N (b) $\bar{T}_p$ as the function of N

Fig. 2. System Scalability

Exp. 2) Performance Measures: $\bar{T}_d$ and $\bar{T}_p$: In this experiment, we investigate the accuracy of the derived performance measures, namely, the average downloading time $\bar{T}_d$ and system throughput $\bar{T}_p$. We set $M = 500$, $\mu = 0.3$, $\gamma = 1.0$, $\rho = 0.5$, $B = 9$ and vary the number of peers in the system. As shown in Fig.2, the BT-like system scales well with the number of peers. Note that the analytical results match well with the simulation results in that there will be a decrease of average downloading time when more peers are in the system. This property is also reported from the real BT-trace data [12]. The nearly linear relationship between the number of peers and the system throughput is also reported in [8].

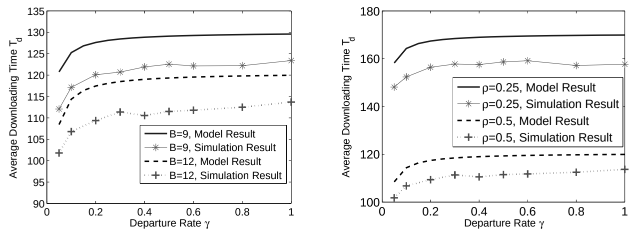
Exp. 3) Sensitivity Analysis: In this set of experiments we investigate the sensitivity of performance measures to various system parameters.

3a) The relationship between T_d and arrival rate λ: For this experiment, we set $M = 500$, $\mu = 0.3$, $\gamma = 1.0$. Fig.3(a) and



(a) For $B = 9$ and 12 (b) For $\rho = 0.25$ and 0.5

Fig. 3. $\bar{T}_d$ as the function of arrival rate λ



(a) For $B = 9$ and 12 (b) For $\rho = 0.25$ and 0.5

Fig. 4. $\bar{T}_d$ as the function of departure rate γ

3(b) illustrates the effect on the average downloading time when we vary the arrival rate λ under different values of B and ρ. Both of these figures show us that when the value of arrival rate becomes large, the average downloading time decreases monotonically and eventually reaches a fixed value when the transmission bandwidth is saturated.

3b) The relationship between T_d and departure rate γ: In this experiment, we set the arrival rate $\lambda = 0.3$, $M = 500$, $\mu = 0.3$ but vary the values of γ. Fig.4(a) is the average downloading time with for $B = 9$ and 12 while Fig.4(b) is the average downloading time for $\rho = 0.25$ and 0.5. These two figures also confirm that by increasing the departure rate γ, the seeder spends less time in the system, hence the average downloading time for peers increases. Notice that when γ is large enough, the rate of deterioration on the file downloading time approaches zero. This implies that even when there is no incentive for peer to be a seeder, the BT-like system can still provide service to peers.

3c) The relationship between T_d and connection probability ρ. From Fig.3(b) and Fig.4(b), we observe that when there are more connections to peers (i.e., ρ is of high value), then the file downloading time actually decreases. From Fig.3(b), we observe that a more highly connected system has lower downloading time, especially when λ is small. As λ increases, the performance difference between different values of ρ diminishes. So for a system with a low arrival rate, high connection probability is important to improve the performance.

3d) The relationship between T_d and bandwidth: In Fig.4(a), the system with more bandwidth has better average downloading time. But in Fig.3(a), we find that for the low arrival rate case, higher transmission bandwidth does not necessary bring better performance. One can achieve better performance when the peer's arrival rate is high because there will be more peers contribute in uploading missing chunks.

V. Conclusion

In this paper, we propose a fluid model based on the stochastic differential equation method in modeling and characterizing the peer evolution of BT-like P2P systems. We obtain the analytical expressions of the average number of seeders and leechers, as well as the average file downloading time and the steady state system throughput. The mathematical model is validated via simulation and the proposed method has a much higher accuracy than the previous reported results [4]. We also quantify the sensitivity of the downloading time to various system parameters such as peers' arrival rate, seeder's departure rate, connection probability and transmission bandwidth. We believe the methodology will provide a better insight in designing the next generation P2P file sharing systems.

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