

# Project: Monte Carlo Simulation

Due: April 21, 2011, 4:30 PM

Instructor: John C.S. Lui

## Note: Deduction Policy for Late Submissions.

Late submissions must be sent to [hongx87@gmail.com](mailto:hongx87@gmail.com)

- Deduct 30% for one day late submission (within 24 hours).
- Deduct 60% for two days late submission (within 48 hours).
- Deduct 100% for more than two days late submission (after 48 hours).

## Note:

- You need to submit the answer of each problem as well as the source code.
- You are only allowed to use Java, C, or C++.
- Make sure that your source code can be compiled correctly, otherwise, you will get 0 point.

1. Estimate the value of  $\pi$  via Monte Carlo simulation. (Reference: tutorial 4)  
Hint: Consider a square and a circle in Figure 1.

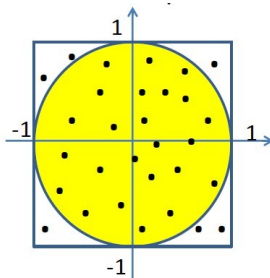


Figure 1: Estimate  $\pi$ .

If we randomly throw  $N$  points in the square, and use  $P$  to denote the number of points which lie in the circle, then we have  $\pi \approx \frac{4P}{N}$ , and we define  $\hat{\pi} = \frac{4P}{N}$ , the error is  $\frac{\hat{\pi} - \pi}{\pi}$ . Run the simulation many times, compute  $\hat{\pi}$  in each time. Draw the error distribution like Figure 2. Specifically, finish the following three steps.

- (a)  $N = 100$ , run 1000 times, estimate  $\pi$  and draw the error distribution.
- (b)  $N = 10000$ , run 1000 times, estimate  $\pi$  and draw the error distribution.
- (c)  $N = 1$  million, run 1000 times, estimate  $\pi$  and draw the error distribution.

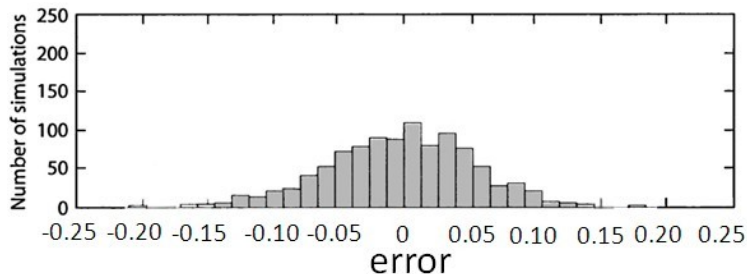


Figure 2: Error distribution.

2. Estimate the probability of straight?

Assume that you have 52 cards, Ace to King with four different colors. Randomly choose five cards from the 52 cards, what is the probability of a straight, e.g., Figure 3? Estimate the probability via Monte Carlo simulation. Hint: run the simulation  $N$  times, in each time, randomly choose five cards and check whether it is a straight. Count the number of times in which the straight appears, which is denoted as  $P$ . Estimate the probability via  $\frac{P}{N}$ . Specifically, finish the following three steps.

- (a)  $N = 1000$ , estimate the probability of straight.
- (b)  $N = 1$  million, estimate the probability of straight.
- (c)  $N = 10$  million, estimate the probability of straight.



Figure 3: Straight

3. **The Appeals Court Paradox:** estimate the probability that the court makes an incorrect decision?

Imagine that a court consists of five judges who meet regularly to vote (independently, of course) on the fate of prisoners. The result of each of the court's deliberations is determined by a simple majority rule, i.e., for a petitioner to be granted or denied a new trial requires three or more votes. Assume that  $A$  votes correctly 95% of the time. Similarly,  $B, C, D,$  and  $E$  vote correctly 95%, 90%, 90%, and 80% of the time. Write a Monte Carlo simulation to estimate the probability that the court makes an incorrect decision? How about  $E$  no longer voting independently, but rather always voting the same as  $A$ ?

Hint: run the simulation  $N$  times, and count the number of times in which the court makes an incorrect decision, which is denoted as  $P$ . Estimate the probability via  $\frac{P}{N}$ . Specifically, finish the following three steps.

- (a)  $N = 1000$ , estimate the probability of incorrect decision.
- (b)  $N = 1$  million, estimate the probability of incorrect decision.
- (c)  $N = 10$  million, estimate the probability of incorrect decision.