Homework # 2
Due: Jan 28, 2011, 4:30 PM

Instructor: John C.S. Lui

Note: Deduction Policy for Late Submissions.

Late submissions **must** be handed to TA'office (Room 120, SHB) by person. We will not handle any late submissions in the homework collection box in 10/F, SHB.

- Deduct 30% for one day late submission (within 24 hours).

- Deduct 60% for two days late submission (within 48 hours).

- Deduct 100% for more than two days late submission (after 48 hours).

1. Four components are inspected and three events are defined as follows:
   A = "All four components are found defective."
   B = "Exactly two components are found to be in proper working order."
   C = "At most three components are found to be defective."
   Interpret the following events:
   (a) $B \bigcup C$.
   (b) $B \bigcap C$.
   (c) $A \bigcup C$.
   (d) $A \bigcap C$.

2. Consider a pool of six I/O buffers. Assume that any buffer is just as likely to be available (or occupied) as any other. Compute the probabilities associated with the following events:
   A = " At least two but no more than five buffers occupied."
   B = "At least three but no more than five buffers occupied."
   C = " All buffers available or an even number of buffers occupied."
   Also determine the probability that at least one of the events A, B, and C occurs.

3. How many even two-digit numbers can be constructed out of the digits 3, 4, 5, 6 and 7? Assume first that you may use the same digit again; then repeat the question, assuming that you may not use a digit more than once.

4. If a three-digit decimal number is chosen at random, find the probability that exactly k digits are ≥ 5, for $0 \leq k \leq 3$.

5. A box with fifteen integrated circuit chips contains five defectives. If a random sample of three chips is drawn, what is the probability that all three are defective?

6. In a party of five persons, compute the probability that at least two have the same birthday (month/day), assuming a 365-day year.

7. A series of n jobs arrive at a computing center with n processors. Assume that each of the n^n possible assignment vectors (processor for job 1, ..., processor for job n) is equally likely. Find the probability that exactly one processor will be idle.

8. Bob needs to reach his sweetheart Alice. As illustrated in figure 1, both of them are located in an m by n city grid. Bob is located in the lower left hand corner while Alice is located in the upper right hand corner. Bob is smart and he only wants to travel through the *shortest path*.
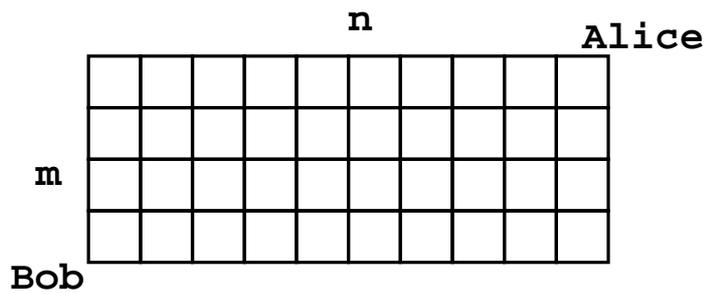

Figure 1: Bob needs to reach Alice

- If $m = 2; n = 3$. How many shortest paths are there for Bob to choose from?
- What is the characteristics of these shortest paths?
- If $m \geq 1$ and $n \geq 1$, how many shortest paths are there in an m by n grid for Bob to choose from?