Week 7 Tutorial Session

1. Give context-free grammars for the following languages. Briefly explain what the rules in your grammar represent. If you use more than one variable, also briefly explain what the variables mean. Justify your answer.

   (a) $L_1 = \{w \in \{0, 1\}^* \mid w \text{ is a palindrome}\}$

      A palindrome is a word that reads the same forward and backward. Examples of palindromes in English are `level` and `madam`.

   (b) $L_2 = \{0^a 1^b \mid a \neq b\}$

   (c) $L_3 = \{w \in \{0, 1\}^* \mid w \text{ contains the same number 0s and 1s}\}$

2. Consider the following context-free grammar G:

$$S \to SS \mid T$$
$$T \to \texttt{a}T\texttt{b} \mid \texttt{ab}$$

   (a) What is the language $L(G)$ of the grammar?

   (b) Show that G is ambiguous.

   (c) Give an equivalent, unambiguous grammar for the same language.

3. (We won't go over this question during the tutorial. This question is here so that students can learn from its solution after the solution is posted.)

   Show that the following language is not context-free by using the pumping lemma for context-free languages.
$$L = \{\texttt{a}^n \texttt{b}^{2n} \texttt{c}^{3n} \mid n \geq 0\}$$