Week 12 Tutorial Session

(1) Consider the language

$$L = \{\langle G_1, G_2\rangle \mid G_1, G_2 \text{ are context-free languages and } L(G_1) = L(G_2)\}$$

(a) Show that L is undecidable.

(b) What is $\overline{L}$? Show that $\overline{L}$ is recognizable.

(c) Show that L is unrecognizable.

(2) Show that the following language is in NP.

$$\textsc{Graph-Isomorphism} = \{\langle G_1, G_2\rangle \mid G_1 \text{ and } G_2 \text{ are isomorphic graphs}\}$$

Two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic if there is a bijection $\varphi : V_1 \to V_2$ mapping vertices of G_1 to vertices of G_2, so that edges and non-edges are preserved, that is $(u, v) \in E_1$ if and only if $(\varphi(u), \varphi(v)) \in E_2$.

(3) In class, we mentioned two definitions of NP. According to the first definition, a language L is in NP if it has a polynomial time verifier V. In other words,

$$x \in L \text{ if and only if there exists } s \text{ such that } V \text{ accepts } \langle x, s\rangle.$$

According to the second definition, a language L is in NP if it is accepted by a nondeterministic polynomial time Turing machine. Here a nondeterministic Turing machine accepts an input x if it accepts x in at least one computation path, and such a machine is polynomial time if all of its computation paths have length bounded by the same polynomial.

Show that these two definitions are equivalent. *Hint:* How is the "candidate solution" s related to the computation paths in a nondeterministic Turing machine M?