CENG4480 Homework 2

Due: Nov. 13, 2018

Q1 The circuit shown in Figure 1 represents a simple 4-bit digital-to-analog converter. Each switch is controlled by the corresponding bit of the digital number if the bit is 1 the switch is up; if the bit is 0 the switch is down. Let the digital number be represented by $b_3b_2b_1b_0$. Please answer the following two questions:

(1) Determine an expression relating v_o to the binary input bits.

(2) Use this converter, design another 4-bit digital-to-analog converter whose output is given by

$$v_o = -\frac{1}{10}(8b_3 + 4b_2 + 2b_1 + b_0)V. \quad (1)$$

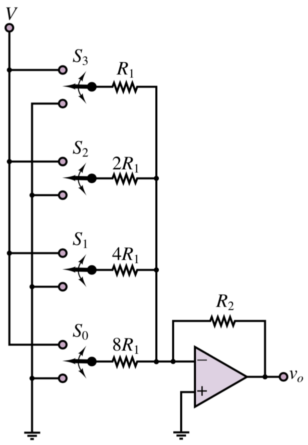

Figure 1: 4-bit DAC.

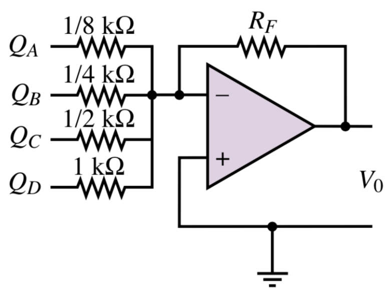

Figure 2: RF DAC.

Q2 For the DAC circuit shown in Figure 2 (using an ideal op-amp), what value of RF will givean output range of $-10 \leq V_0 \leq 0V$? Assume that logic $0 = 0V$ and logic $1 = 5V$.

Q3 A simple Infra-Red Sensor system to detect passing human is presented as in Figure 3. A and B are IR Sensors which will generate different output voltages for different infra-red intensity, and higher voltage level corresponds to high light intensity.

(1) Explain how this system works for counting passing pedestrians.

(2) To increase counting accuracy, usually B is covered with materials that can reflect infra-red light. Explain why.

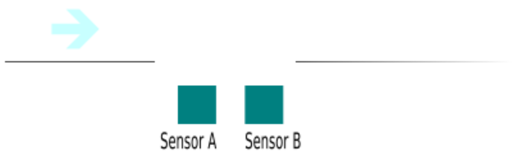

Figure 3: IR-System.

Q4 Exemplify the working principles of sensors that measure: (1) Flow; (2) Temperature; (3) Pressure; (4) Motion; (5) Liquid Level.

Q5 Briefly describe how PID affects motor control.

Q6 Given a linear system

$$\begin{cases} \boldsymbol{x}_t = \boldsymbol{A}_{t-1}\boldsymbol{x}_{t-1} + \boldsymbol{\omega}_{t-1}, \\ \boldsymbol{z}_t = \boldsymbol{B}_t\boldsymbol{x}_t + \boldsymbol{v}_t, \\ \boldsymbol{v}_t = \boldsymbol{C}_{t-1}\boldsymbol{v}_{t-1} + \boldsymbol{n}_{t-1}, \end{cases} \tag{2}$$

where $\boldsymbol{\omega}_t$ and $\boldsymbol{n}_t$ are independent and obey Gaussian distribution zero-mean and covariance $\boldsymbol{Q}_t$ and $\boldsymbol{R}_t$, respectively. Please give the estimate equation and measurement equation of the system.

Q7 Given two Gaussian distributions $N(x_0; \mu_0, \sigma_0)$ and $N(x_1; \mu_1, \sigma_1)$, try to give the expectation and variance of a new distribution which is the product of these two Gaussian distributions.

Q8 For the 4-bit R-2R DAC, calculate V_0 in terms of $V_{b,0} - V_{b,4}$ if V_{ref} is grounded (Figure 4).

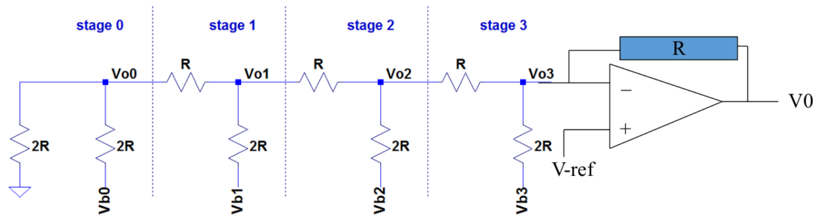

Figure 4: R-2R DAC.

Q9 (Discrete-time random walk) Suppose that we wish to estimate the position of a particle that is undergoing a random walk in one dimension (i.e., along a line). We model the position of the particle as

$$x[k+1] = x[k] + u[k],$$

where x is the position of the particle and u is a white noise processes with $E\{u[i]\} = 0$ and $E\{u[i]u[j]\} = R_u\theta(i-j)$. We assume that we can measure $\boldsymbol{x}$ subject to additive, zero-mean, Gaussian white noise with covariance 1.

Construct a Kalman filter to estimate the position of the particle given the noisy measurements of its position. Compute the steady-state expected value and covariance of the error of your estimate.