CENG4480 Homework 1

Due: Oct. 28, 2019

Q1 (10%) Given a circuit as shown in Figure 1, input $V_I = 1V$, the resistors $R_1 = R_2 = 10k\Omega$, the variable resistor $R_p = 20k\Omega$.

1. when the sliding of the variable resistor is connected to A, calculate V_O.
2. when the sliding of the variable resistor is connected to B, calculate V_O.
3. when the sliding of the variable resistor is connected to C (midpoint), calculate V_O.

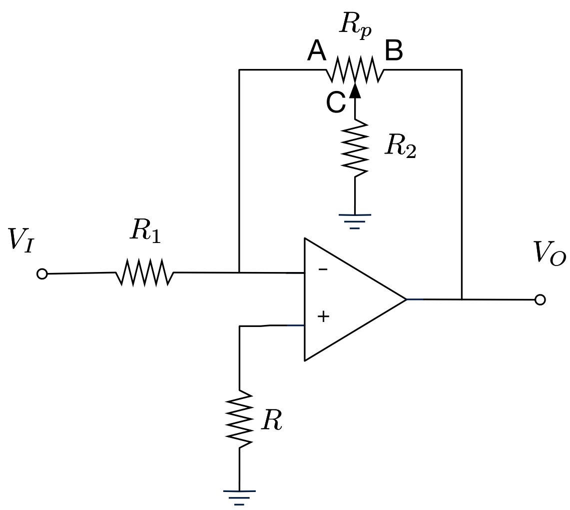


Figure 1: Q1 circuit

A1

$$V_O = -2V \tag{1}$$

Details are shown in Figure 2.

$$V_O = -2V \tag{2}$$

Details are shown in Figure 3.

$$V_O = -3V \tag{3}$$

Details are shown in Figure 4.

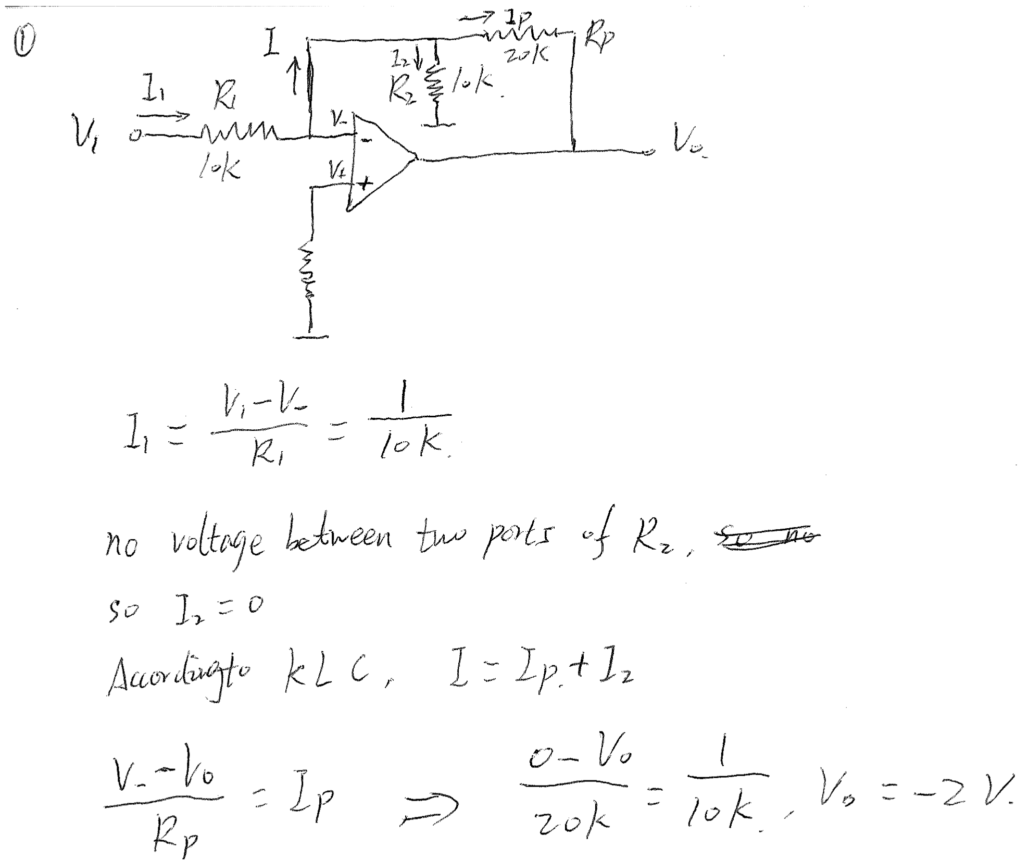
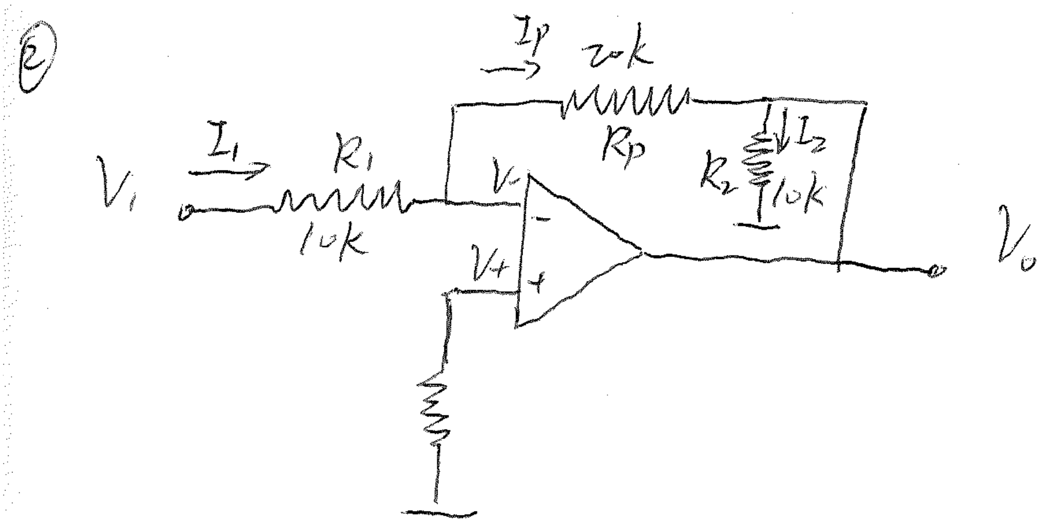


Figure 2: A1-1 solution



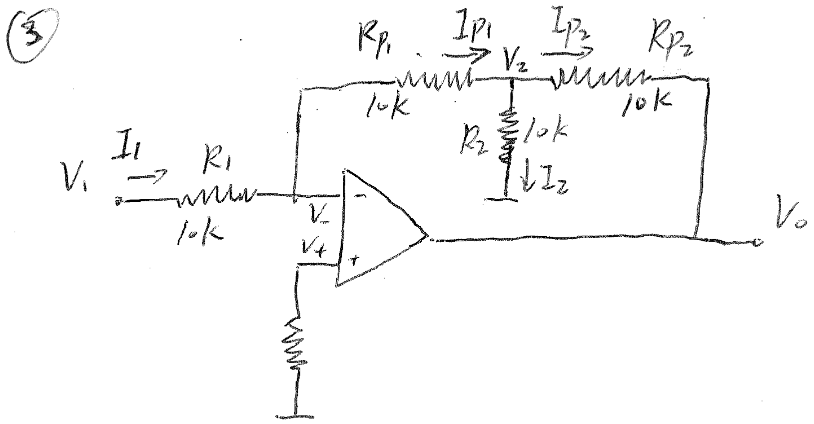
$$I_1 = \frac{V_1 - V_-}{R_1} = \frac{1}{10k}$$

$$I_1 = I_p$$

$$\frac{V_- - V_o}{R_p} = I_p$$

$$V_o = -I_p R_p = -\frac{1}{10k} 20k = -2\ V$$

Figure 3: A1-2 solution



$$I_1 = I_{p1}$$

$$\Rightarrow \frac{V_1 - V_-}{R_1} = \frac{V_- - V_2}{R_{p1}} \Rightarrow \begin{cases} V_2 = -1V \\ I_1 = I_{p1} = \frac{1}{10k} \end{cases}$$

$$\begin{cases} I_{p1} = I_2 + I_{p2} \\ I_2 = \frac{V_2}{R_2} = \frac{-1}{10k} \end{cases} \Rightarrow \begin{cases} I_{p2} = \frac{2}{10k} \\ I_{p2} = \frac{V_2 - V_o}{R_{p2}} \end{cases} \Rightarrow V_o = -3V$$

Figure 4: A1-3 solution

Q2 (10%) Given a differential circuit as shown in Figure 5, determine the mathematical relationship among V_O, V_{I1} and V_{I2}.

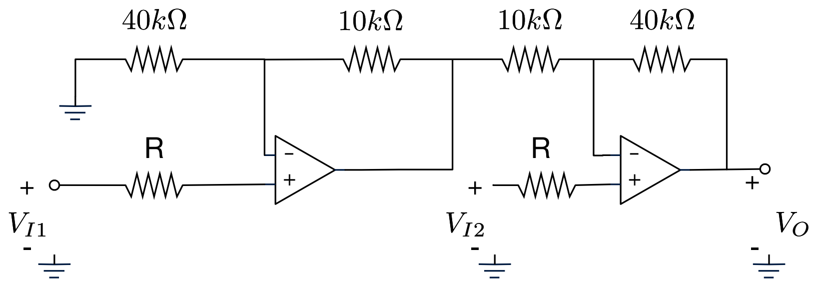


Figure 5: Q2 circuit

A2 $V_O = 5(V_{I2} - V_{I1})$

Q3 (10%) Given the inverting amplifier as shown in Figure 6, its supply voltage is $\pm 15V$.

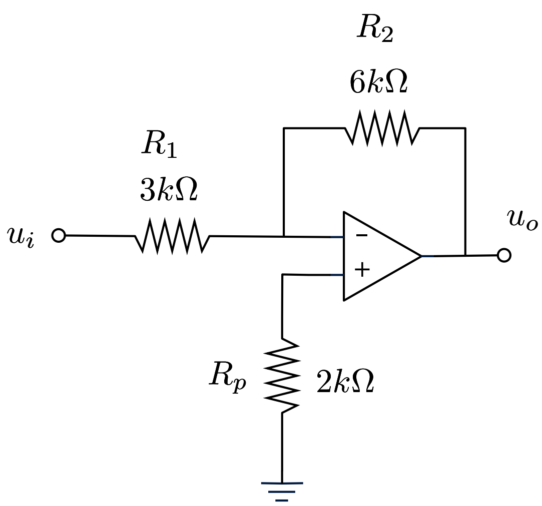


Figure 6: Inverting Amplifier.

1. Compute and sketch transmission curve between u_i and u_o.
2. The input signal is given to be $u_i = 5sin\omega t(V)$, sketch the waveform of u_o.

A3 A3 solution is shown in Figure 7:

Q4 (15%) A differential integrator is shown in Figure 8.

1. Determine the relationship among u_{i1}, u_{i2} and u_o.
2. If we want $u_o = 0V$ when $u_{i2} = 1V$, determine u_{i1}
3. When $t = 0$, $u_{i2} = 1V$, $u_{i1} = 0V$, $u_o = 0V$, determine u_o when $t = 10s$.

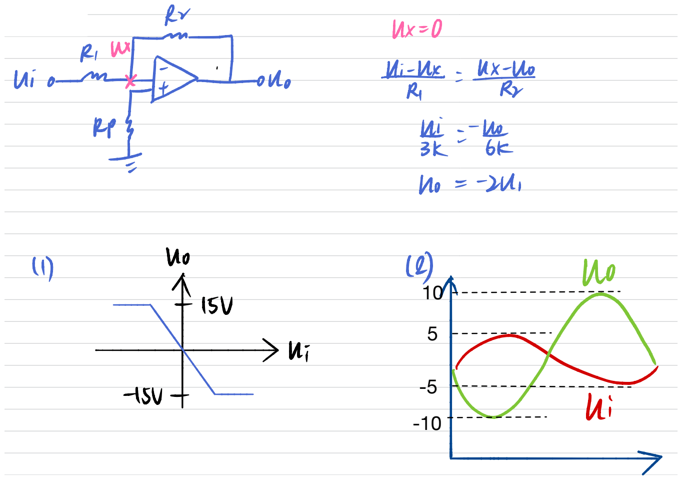


Figure 7: A3 solution

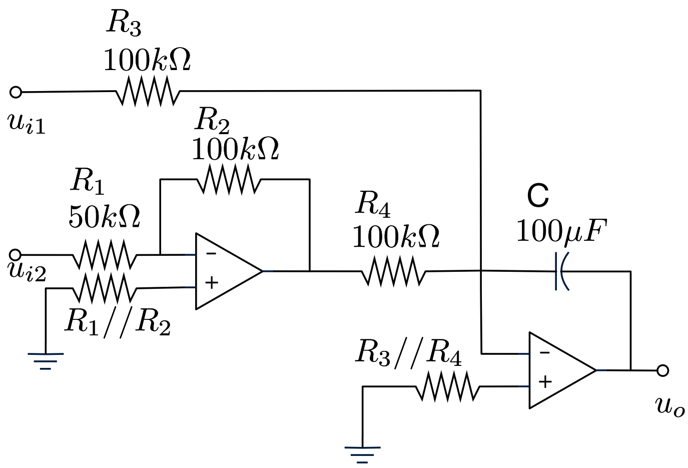


Figure 8: A differential integrator.

A4 Let left op amp output is u_{o2}, according to the inverting amp Gain shown in page 30 of Lecture 02, we have

$$u_{o2} = -\frac{R_2}{R_1}u_{i2} = -2u_{i2} \tag{4}$$

According to the the relationship between input and output of Integrator shown in page 14 of Lecture 03, we have

$$u_o(t) = -\frac{1}{C}\int_0^t \frac{u_{i1}(t)}{R_3}dt - \frac{1}{C}\int_0^t \frac{u_{o2}(t)}{R_4}dt \tag{5}$$

$$= \frac{1}{R_3C}\int_0^t (2u_{i2}(t) - u_{i1}(t))dt \tag{6}$$

initial voltage between two ports of capacitor $U_{C(0)} = 0$, we have

$$u_o(t) = \frac{1}{R_3 C}\int_0^t (2u_{i2}(t) - u_{i1}(t))dt \tag{7}$$
$$= \frac{1}{100\times 10^3 \times 100 \times 10^{-6}}\int_0^t (2u_{i2}(t) - u_{i1}(t))dt \tag{8}$$
$$= 0.1\int_0^t (2u_{i2}(t) - u_{i1}(t))dt \tag{9}$$

$$u_o = 0.1\int_0^t (2u_{i2}(t) - u_{i1}(t))dt = 0 \tag{10}$$

$$u_{i1}(t) = 2u_{i2}(t) = 2V \tag{11}$$

$$u_o(t) = 0.1\int_0^t (2u_{i2}(t) - u_{i1}(t))dt \tag{12}$$
$$= 0.1\int_0^t (2\times 1)dt \tag{13}$$
$$= 0.2t \tag{14}$$
$$= 0.2\times 10 \tag{15}$$
$$= 2V \tag{16}$$

Q5 (20%) Given a low-pass filter as shown in Figure 9.

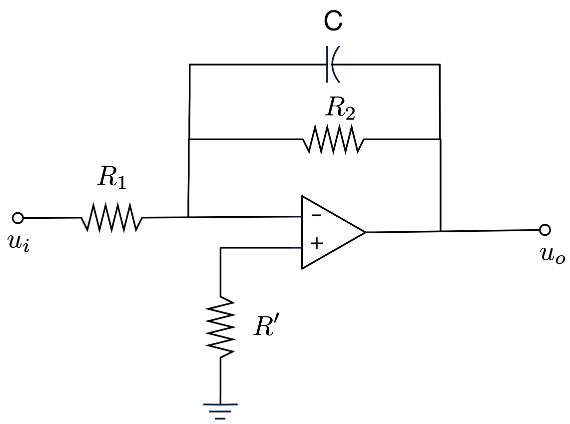


Figure 9: A low-pass filter.

1. If $R_1 = 10K\Omega$, $R_2 = 100K\Omega$, determine low-frequency gain $A_u(dB)$;
2. If cutoff frequency $f_c = 5Hz$, determine C value.

A5

$$u_o(j\omega) = -\frac{R_2//\frac{1}{j\omega C}}{R_1}u_i(j\omega) \tag{17}$$

So

$$A_u(j\omega) = \frac{u_o(j\omega)}{u_i(j\omega)} = -\frac{R_2}{R_1}\frac{1}{1 + j\omega R_2 C} = \frac{A_{u_o}}{1 + j\frac{\omega}{\omega_c}} \tag{18}$$

where $A_{u_o} = -\frac{R_2}{R_1}$ is low-frequency gain, $\omega_c = \frac{1}{R_2 C}$ is cutoff angular frequency. If $R_1 = 10K\Omega$, $R_2 = 100K\Omega$, low-frequency gain

$$20\log\frac{R_2}{R_1} = 20\log\frac{100}{10} = 20dB \tag{19}$$

cutoff frequency:

$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi R_2 C} = \frac{1}{2\pi \times 100 \times 10^3 \times C} = 5Hz \tag{20}$$

so

$$C = \frac{1}{2\pi \times 100 \times 10^3 \times f_c} = \frac{1}{2\pi \times 100 \times 10^3 \times 5} = 3.18 \times 10^{-7}F = 0.318\mu F \tag{21}$$

Q6 (10%) Determine the output voltage (i.e. the mathematical expression of $u_o(t)$) for the differentiator circuit of Figure 10 if the input is a triangular wave of amplitude $\pm 0.2V$ and frequency $1Kz$. Assume $C = 0.1\mu F$, $R_1 = 200\Omega$, $R_2 = 10k\Omega$, $R_3 = 1\Omega$, $R_p = 1k\Omega$ and ideal op-amp. The triangular wave starts at $t = 0$ and therefore $u_o(0) = 0$.

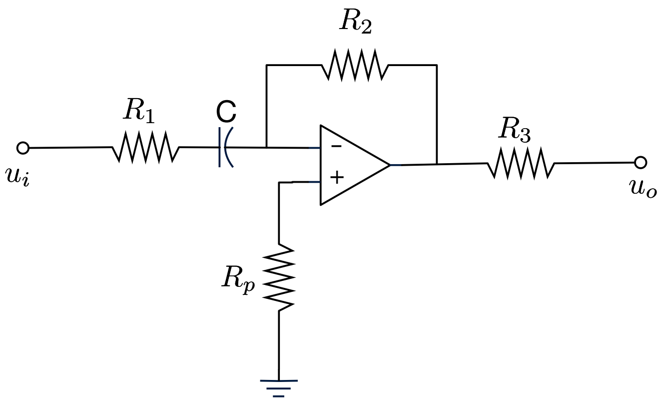


Figure 10: A differentiator circuit.

A6 We assume that voltmeter is used to measure the output voltage u_o. In general, the resistance value of voltmeter R_{VM} is about $1G\Omega$ [1]. In addition, R_3 is in series with R_{VM} in the circuit. The relationship between u_o and u_{op-o} is $u_o(R_{VM} + R_3) = u_{op-o}R_{VM}$. Furthermore, $R_{VM} >> R_3$. So $u_o \approx u_{op-o}$.

We set $s = jw$, then $u_{opi-} = u_{opi+} = 0$. $I_{R_1} = u_i/Z$, where $Z = R_1 + \frac{1}{sC}$. $I_{R_2} = -u_o/R_2$. According to KCL, $I_{R_1} = I_{R_2}$. Then $\frac{sCu_{in}}{1+sR_1C} = -\frac{u_o}{R_2}$. Then $u_o = -\frac{R_2 sCu_{in}}{1+sR_1C}$. Since $1 >> R_1C$, $u_o \approx -R_2 sCu_{in} = -R_2 j\omega Cu_{in}$, which is the totally same the relationship between input and output of differentiator circuit shown in the page 15 of Lecture 03 by using impedance representation shown in the page 11 of Lecture 03.

[1] `https://en.wikipedia.org/wiki/Voltmeter`

According to the page 15 of Lecture 03, we have

$$u_o \approx -R_2 C \frac{du_{in}(t)}{dt}, \tag{22}$$

where for the triangular wave input, $\frac{du_{in}(t)}{dt} = \pm(0.2 - (-0.2))/0.5 = \pm 0.8$.

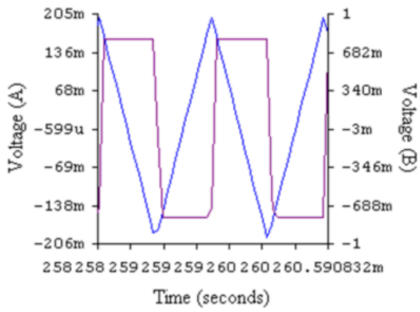


Figure 11: waveform.

Q7 (10%) An ADC is used to sample an analog signal.

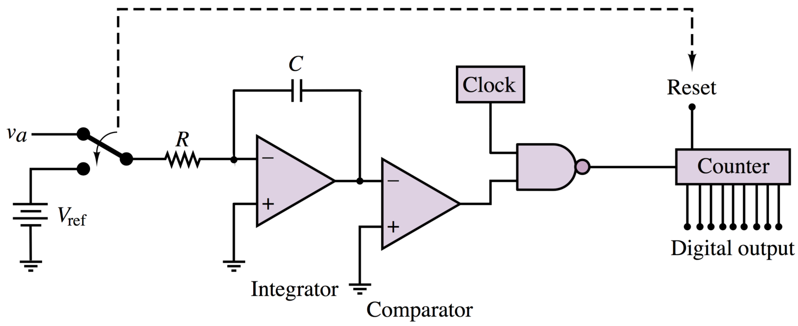


Figure 12: Integrating ADC.

1. If the maximum frequency of the analog signal is $10kHz$, determine the minimum sampling frequency.
2. As shown in Figure 12, if the ADC is integrating ADC with 10 bits and clock frequency is $1MHz$, determine the maximum conversion frequency.

A7 $20kHz$ conversion time $T = 2^{n+1}T_c$, $T_c = 1/10^6$, $T = 2.048ms$.

Q8 (10%) Let us consider the Schmitt Trigger shown in Figure 13.

1. Due to the manufacturing defects, a parasitic resister R_3 occurs between the output node and ground, calculate the reference voltages.
2. If the parasitic device is a capacitor C, sketch v_{out} versus v_{in}. Label the key coordinates on the curve.

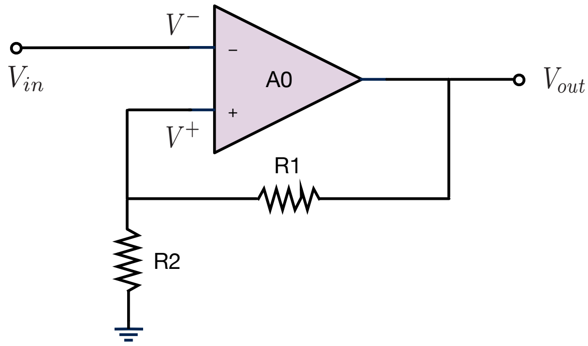


Figure 13: Schmitt Trigger.

A8 1. According to the properties of comparator, when v_{in} is small, $v_{out} = v_{sat}$ and

$$\frac{v^+}{R_2} = \frac{v_{out} - v^+}{R_1}, \tag{23}$$

i.e.,

$$v^+ = \frac{R_2}{R_1 + R_2} v_{sat}. \tag{24}$$

Similarly, if v_{in} is large, we have

$$v^+ = -\frac{R_2}{R_1 + R_2} v_{sat}. \tag{25}$$

Therefore two reference voltages are given by $\frac{R_2}{R_1+R_2} v_{sat}$ and $-\frac{R_2}{R_1+R_2} v_{sat}$.

2. v_{out} start to change when v_{in} reaches references above. However, due to the existing capacitor, voltage cannot change immediately (Changes fast, then slowly).

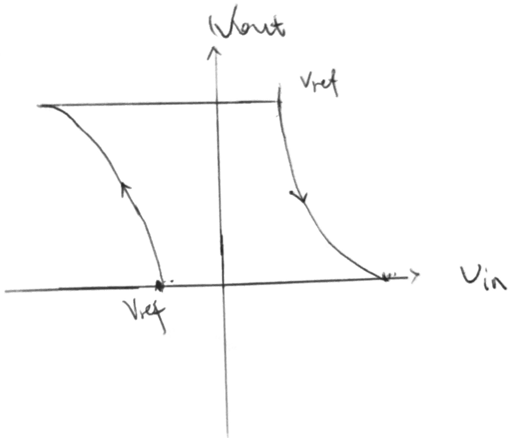


Figure 14: A8(2).