

## CSCI3160: Special Exercise Set 8

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**Problem 1.** Prove: all the SCCs of a directed simple graph are mutually disjoint.

**Problem 2.** Let  $G = (V, E)$  be a directed simple graph and  $G^{scc}$  be the SCC graph defined in our lecture. Let  $S_1$  and  $S_2$  be two SCCs of  $G$ . Prove: if  $S_1$  cannot reach  $S_2$  in  $G^{scc}$ , then no vertex of  $S_1$  can reach any vertex of  $S_2$  in  $G$ .

**Problem 3.** Prove:  $G$  and  $G^{rev}$  have the same SCCs.

**Problem 4.** Prof. Goofy proposes his own SCC algorithm:

- Step 1: Perform DFS on the input graph  $G$  and compute a label for each vertex (just like Step 1 of our algorithm).
- Step 2: Perform another DFS on  $G$  (note: not on  $G^{rev}$ ) subject to the following rules:
  - Start the first DFS from the vertex with the smallest label.
  - Whenever a restart is needed, do so from the white vertex with the smallest label.

Give a counterexample to prove that Prof. Goofy is wrong.

**Problem 5.** Design an algorithm to generate  $G^{scc}$  from  $G = (V, E)$  in  $O(|V| + |E|)$  time.