

CSCI3160: Special Exercise Set 7

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Problem 1. Let x be a string of length n , and y a string of length m . Define $opt(i, j)$ to be the length of an LCS of $x[1 : i]$ and $y[1 : j]$ for $i \in [0, n]$ and $j \in [0, m]$. Compute the values of all possible (i, j) for $x = 10010101$ and $y = (010110110)$.

Problem 2. Find an LCS of x and y given in Problem 1.

Problem 3. Given a string s of length n , stored in an array of characters, we call $s[i : j]$ a *substring* of s , for all pairs of i, j satisfying $1 \leq i \leq j \leq n$. Let x be a string of length n , and y a string of length m . Design an algorithm to find a longest common substring of x and y in $O(nm)$ time.

Problem 4*. Let M be an $n \times n$ matrix where each cell $M[i, j]$ stores a distinct integer, for all $i \in [1, n]$ and $j \in [1, n]$. Define a *path* of length $\ell \geq 1$ to be a sequence of ℓ cells $M[i_1, j_1], M[i_2, j_2], \dots, M[i_\ell, j_\ell]$ satisfying both conditions below:

- for each $k \in [2, \ell]$, $M[i_{k-1}, j_{k-1}]$ and $M[i_k, j_k]$ are neighboring cells (this means the former cell is above, below, to the left of, or to the right of the latter cell);
- for each $k \in [2, \ell]$, $M[i_{k-1}, j_{k-1}] < M[i_k, j_k]$.

Design an algorithm that finds a path of the maximum length in $O(n^2 \log n)$ time.

(Hint 1: Find the length of longest paths starting from each cell)

(Hint 2: To choose a topological order, sort all the cells)

Problem 5.** Improve the running time of your solution to Problem 4 to $O(n^2)$.

(Hint: What is the dependency graph among the cells?)