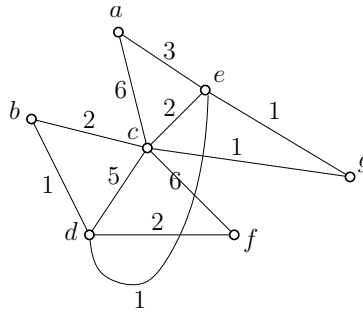


## CSCI3160: Special Exercise Set 4

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**Problem 1.** Let  $T$  be a tree. Prove: for any two distinct nodes  $u, v$  in the tree, there exists one and exactly one simple path from  $u$  to  $v$  (a simple path is a path where no vertex appears twice).

**Problem 2.** Consider the weighted undirected graph below.



Suppose that we run Prim's algorithm to find a minimum spanning tree (MST) of this graph. Explain the order of edges picked by the algorithm.

**Problem 3.** Consider again the execution of Prim's algorithm in Problem 2. Indicate how the cross edges change as Prim's algorithm runs.

**Problem 4 (The Cut-Property)** Let  $G = (V, E)$  be an undirected graph where each edge in  $E$  is associated with a positive weight. Consider any non-empty subset  $S \subset V$ . An edge  $\{u, v\}$  in  $E$  is an  $S$ -cross edge if  $u \in S$  but  $v \notin S$ . Prove: if  $e$  is an  $S$ -cross edge that has the minimum weight among all  $S$ -cross edges,  $e$  must belong to some MST of  $G$ .