The RAM Computation Model

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This is not a programming course.

Main take-away message from this course

Computer science is a branch of mathematics with its art reflected in the beauty of algorithms.

- Programming knowledge is not necessary to study algorithms.

Many people believe that this branch holds the future of mankind.

In mathematics (and hence, computer science) everything—including every term and symbol—must be rigorous.

Computer science is a subject where we

1. first define a **computation model**, which is a simple yet accurate abstraction of a computing machine;
2. then slowly build up a theory in this model from scratch.

The Random Access Machine (RAM) model

A machine has a **memory** and a **CPU**.

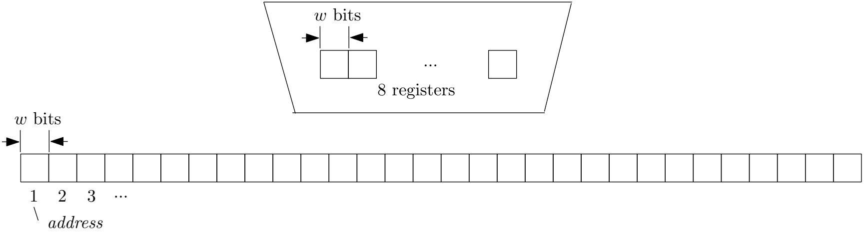
Memory

- An infinite sequence of **cells**, each of which contains the same number w of bits.
- Every cell has an **address**: the first cell of memory has address 1, the second cell 2, and so on.

The Random Access Machine (RAM) model

CPU

- Contains a fixed number—8 in this course—of **registers**, each of which has w bits (i.e., same as a memory cell).



The Random Access Machine (RAM) model

CPU

- Can do the following **atomic operations**:
  1. **(Register (Re-)Initialization)**
     Set a register to a fixed value (e.g., 0, -1, 100, etc.), or to the content of another register.

The Random Access Machine (RAM) model

CPU

- Can do the following **atomic operations**:

  2. **(Arithmetic)**
     Take the integers a, b stored in two registers, calculate one of the following and store the result in a register:
     - $a + b$, $a - b$, $a \cdot b$, and a/b.

> Note: a/b is "integer division", which returns an integer. For example, $6/3 = 2$ and $5/3 = 1$.

The Random Access Machine (RAM) model

CPU

- Can do the following **atomic operations**:

  3. **(Comparison/Branching)**
     Take the integers a, b stored in two registers, compare them, and learn which of the following is true:
     - $a < b$, $a = b$, $a > b$.

The Random Access Machine (RAM) model

CPU

- Can do the following **atomic operations**:

  4. **(Memory Access)**
     Take a memory address A currently stored in a register. Do one of the following:
     - Read the content of the memory cell with address A into a designated register (overwriting the bits there).
     - Write the content of a designated register into the memory cell with address A (overwriting the bits there).

The Random Access Machine (RAM) model

CPU

- Can do the following **atomic operations**:

  5. **(Randomness)**
     - RANDOM(x, y): Given integers x and y (satisfying $x \leq y$), this operation returns an integer chosen uniformly at random in $[x, y]$, and places the random integer in a register.

The Random Access Machine (RAM) model

An **execution** is a sequence of atomic operations.

Its **cost** (also called its **running time**, or simply, **time**) is the length of the sequence, namely, the number of atomic operations.

The Random Access Machine (RAM) model

A **word** is a sequence of w bits, where w is called the **word length**.

- In other words, each memory cell and CPU register store a word.

Unless otherwise stated, you do not need to pay attention to the value of w in this course.

Algorithm

- An **input** refers to the initial state of the registers and the memory before an execution starts.
- An **algorithm** is a piece of description that, given an input, can be utilized to **unambiguously** produce a sequence of atomic operations, namely, the **execution** of the algorithm.
    - In other words, it should be always clear what the next atomic operation should be, given the outcome of all the previous atomic operations.
- The **cost** of an algorithm on an input is the length of its execution on that input (i.e., the number of atomic operations required).

Deterministic Algorithms vs. Random Algorithms

An algorithm is **deterministic** if it never invokes the atomic operation RANDOM. Otherwise, the algorithm is **randomized**.

On the same input, the cost of a deterministic algorithm is a fixed integer—it remains the same every time you execute the algorithm.

The cost of a randomized algorithm, however, is a random variable. Even on the same input, the cost may change each time the algorithm is executed.

Example

```
1. do
2.     r = RANDOM(0, 1)
3. until r = 1
```

How many times would Line 2 be executed? The answer is—"we don't know" (in fact, the line may be executed an infinite number of times)! Every time the above "algorithm" is executed, it may produce a new sequence of atomic operations.

Expected Cost of a Randomized Algorithm

Let X be a random variable that equals the cost of an algorithm on an input. The **expected cost** of the algorithm on the input is the expectation of X.