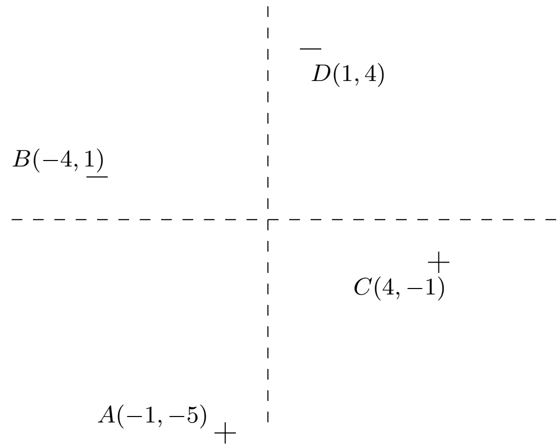
CMSC5724: Exercise List 5

Answer all the problems below based on the following set P of points A, B, C and D:



where "+" represents label 1 and "-" represents label -1.

Problem 1. What is the margin of the separation line $\ell : -x - 5y = 0$?

Problem 2. Run Margin Perceptron on P with $\gamma_{guess} = 0.1$, and give the equation of the line that is maintained by the algorithm at the end of each iteration.

Problem 3. Same as the previous problem but with $\gamma_{guess} = 4/\sqrt{26}$.

Problem 4. Give an instance of quadratic programming to find an origin-passing separation plane with the maximum margin.

Problem 5. Consider the following instance of quadratic programming in $\mathbb{R}^d$:

$$\text{minimize } |\boldsymbol{w}| \text{ subject to}$$
$$\boldsymbol{w} \cdot \boldsymbol{p}_i \geq 1 \text{ for each } i \in [1, n]$$

where $\boldsymbol{p}_1, ..., \boldsymbol{p}_n$ are n given points in $\mathbb{R}^d$. Prove: if an optimal $\boldsymbol{w}$ exists, there must exist at least one $i \in [1, n]$ such that $\boldsymbol{w} \cdot \boldsymbol{p}_i = 1$.

Problem 6. Let γ_{opt} be the maximum margin of an origin-passing separation plane on a set P of points. Denote by R the largest distance from a point in P to the origin.

Suppose that, given a value γ, margin Perceptron ensures the following:

- if it terminates, it definitely returns a separation plane with margin at least $\alpha \cdot \gamma$, where α is an arbitrary constant less than 1;
- if $\gamma \leq \gamma_{opt}$, it definitely terminates after at most $c \cdot R^2/\gamma^2$ corrections, for some constant (which depends on α).

Design an algorithm to find a separation plane with margin at least $\alpha \cdot \beta \cdot \gamma_{opt}$ after $O(R^2/\gamma_{opt}^2)$ corrections in total, where β can be any constant less than 1.