Exercises

Problem 1. Let R be a set of axis-parallel rectangles and P be a set of points, all in $\mathbb{R}^d$ where the dimensionality d is a fixed constant. Set $n = |R| + |P|$. Describe an $O(n \operatorname{polylog} n)$-time algorithm to report, for every rectangle $r \in R$, the number of points in P covered by r.

Problem 2 (sorted dominance screening in 2D). Let P and Q each be a set of points in $\mathbb{R}^2$. Set $n = |P| + |Q|$. Suppose that both P and Q have been sorted by x-coordinate. Describe an $O(n)$-time algorithm to find all the points $q \in Q$ such that q is not dominated by any point in P.

Problem 3* (maxima in 3D). Let P be a set of n points in $\mathbb{R}^3$. Describe an algorithm to find all the maximal points of P in $O(n \log n)$ time.

(Hint: Try to use the result of Problem 2. Sort all the points by y-coordinate first. Still recurse by partitioning the x-coordinates but maintain the y-ordering for each sub-problem.)

Problem 4 (sorted intervals-contain-points). Let R be a set of intervals in the form $[a, b]$, where a and b are real values; these intervals have been sorted by a. Let P be a set of real values in $\mathbb{R}$ which have also been sorted. Describe an algorithm to report all the $(r, p) \in R \times P$ such that r covers p. Your algorithm should run in $O(n + k)$ time where $n = |R| + |P|$ and k is the number of pairs reported.

Problem 5* (rectangles-contain-points in 2D). Let R be a set of axis-parallel rectangles and P be a set of points, all in $\mathbb{R}^2$. We want to report all the $(r, p) \in R \times P$ such that r covers p. Describe an algorithm to do so in $O(n \log n + k)$ time.

(Hint: as in Problem 3, maintain an appropriate y-ordering while doing the recursion by partitioning the x-dimension. Use the result of Problem 4.)