Mathematics and Life: Mathematical Literacy of our Students in HKPISA 2003

Law Huk Yuen

Department of Curriculum and Instruction, CUHK

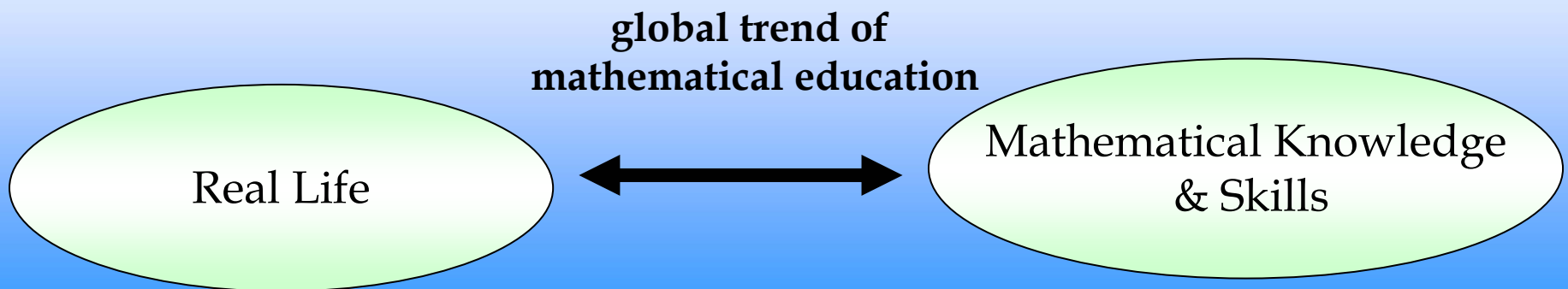*13 December 2004*

Profile

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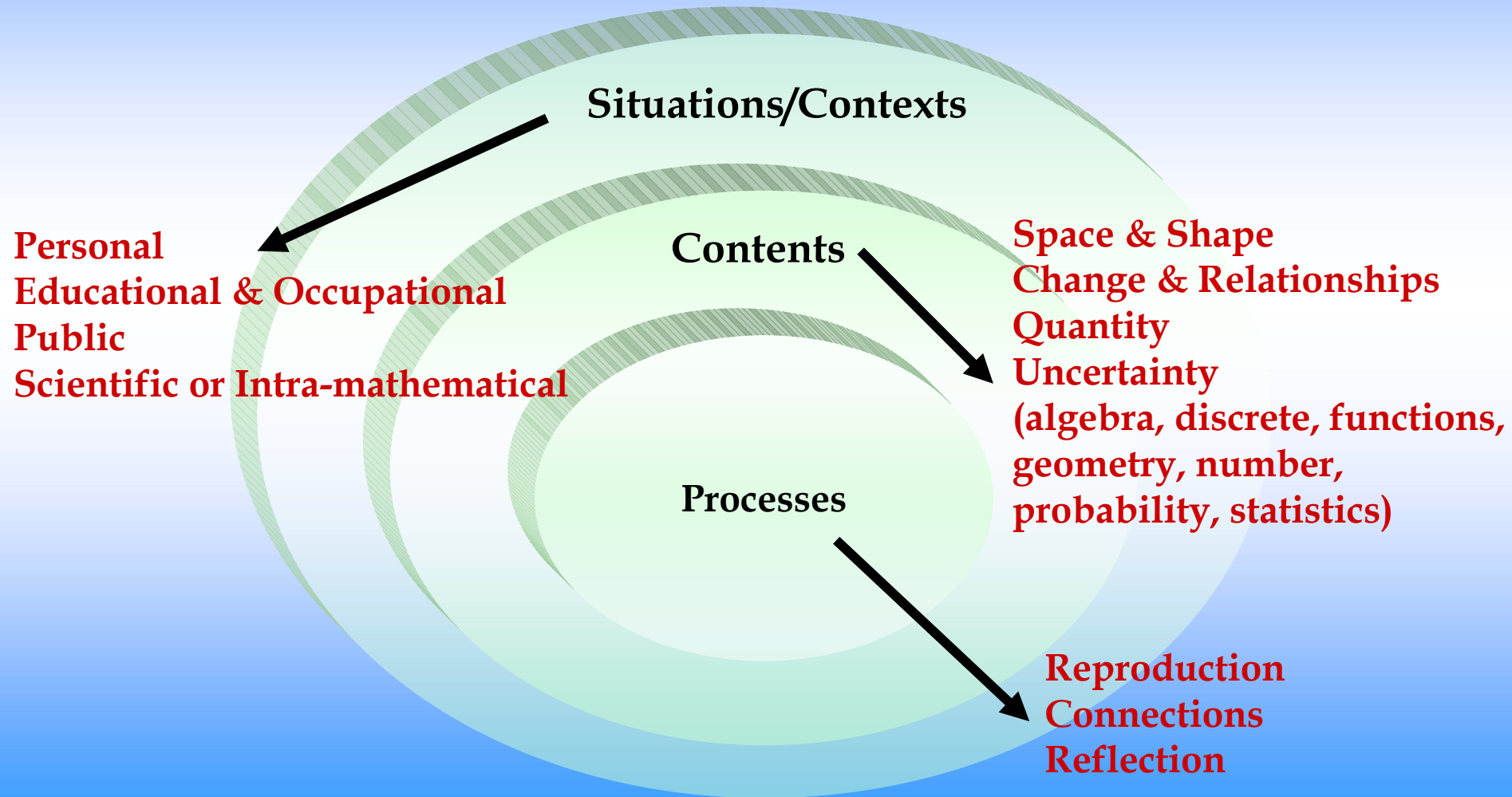
What is Mathematical Literacy?

- *"Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen."* (adapted from OECD/PISA 2003)

- Focus on the capacity of our 15-year-old students to apply their mathematical knowledge and skills across different situations and contexts in daily life, rather than their performance on formal mathematical curriculum.

global trend of mathematical education

Real Life ⟷ Mathematical Knowledge & Skills

Assessment Framework of Mathematics in PISA 2003

- The three assessing dimensions:

Situations/Contexts

Contents

Processes

Personal
Educational & Occupational
Public
Scientific or Intra-mathematical

Space & Shape
Change & Relationships
Quantity
Uncertainty
(algebra, discrete, functions, geometry, number, probability, statistics)

Reproduction
**Connections**
Reflection

Assessment Framework of Mathematics in PISA 2003 (cont'd)

- The distribution of mathematical items (85 in total) by these three dimensions:

Dimensions	Number of Items	Percentages (%)
Contents		
Space & Shape	20	23.5%
Change & Relationships	22	25.9%
Quantity	23	27.1%
Uncertainty	20	23.5%
Processes		
Reproduction	26	30.6%
Connections	40	47.1%
Reflection	19	22.4%
Situations/Contexts		
Personal	18	21.2%
Educational & Occupational	20	23.5%
Public	29	34.1%
Scientific or Intra-mathematical	18	21.2%

Proficiency Levels in Mathematical Literacy
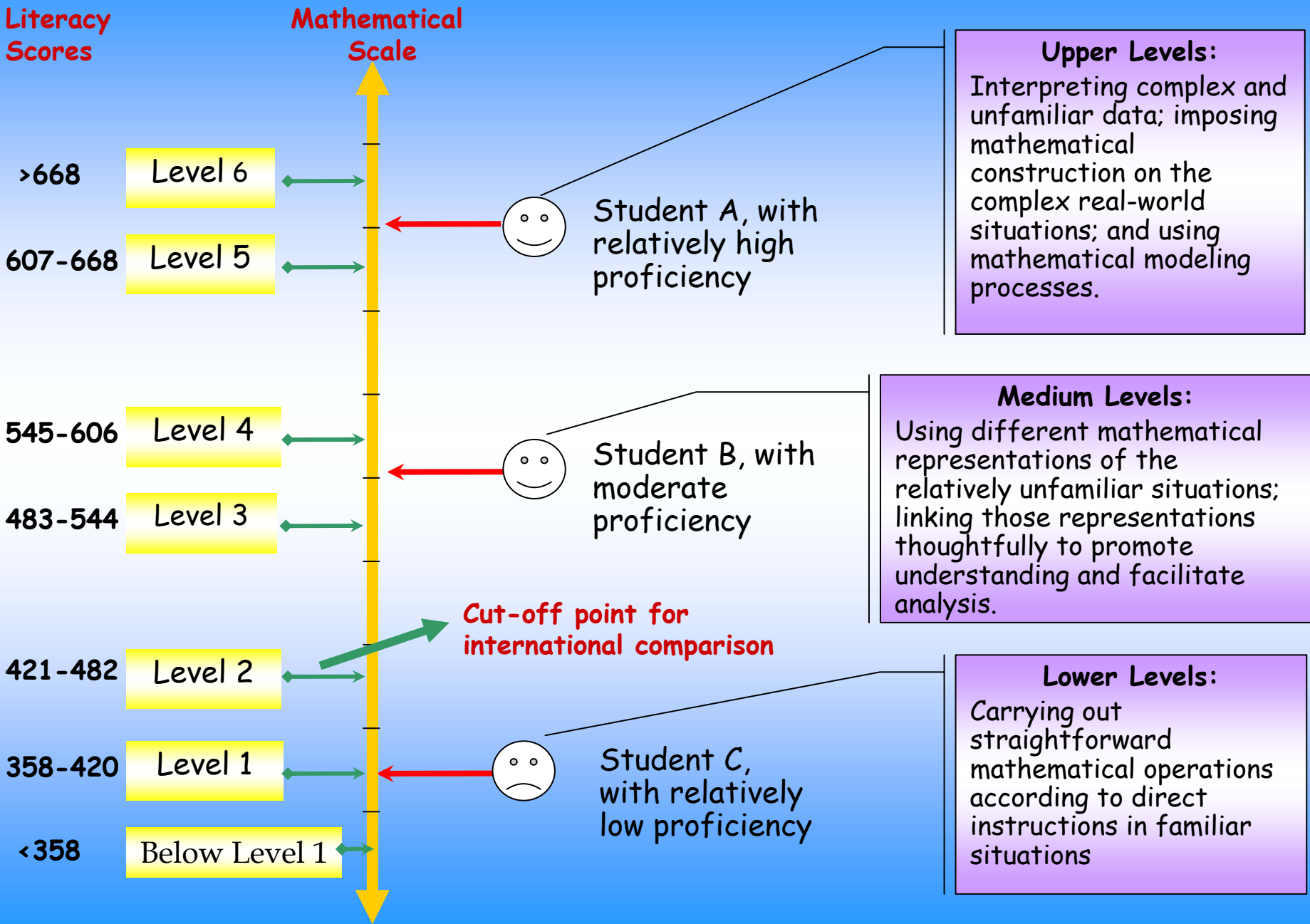
Sample Task 1—兌換匯率(Exchange Rate)

- 引文:

  來自新加坡的美玲正準備到南非三個月當交換生。她需要把一些新加坡元(SGD)兌換為南非鎊(ZAR)。

  題目 1:

  美玲得知新加坡元與南非鎊的兌換匯率為：

  $$1 \ SGD = 4.2 \ ZAR$$

  美玲以此匯率把3000新加坡元兌換成南非鎊。

  那麼，美玲換得多少南非鎊呢？

Quantity/Public/Reproduction/
Short Response

HK Mean % Correct: 89.13 %
OECD Mean % Correct: 79.66 %

Sample Task 1—兌換匯率 (Exchange Rate) (cont'd)

題目 2:

三個月後，美玲返回新加坡，剩下3 900 ZAR。她把南非鍰兌換回新加坡元，發覺兌換匯率已轉為：

$$1 \ SGD = 4.0 \ ZAR$$

那麼，美玲換得多少新加坡元呢？

Quantity/Public/Reproduction/
Short Response

HK Mean % Correct: 88.04 %
OECD Mean % Correct: 73.86 %

題目3:

在這三個月期間，兌換匯率由每一SGD兌4.2轉至4.0 ZAR。

當美玲把南非鍰兌換回新加坡元時，兌換匯率由4.2 ZAR轉至現時的4.0 ZAR，這是否有利於美玲呢？請提供一個解釋以支持你的答案。

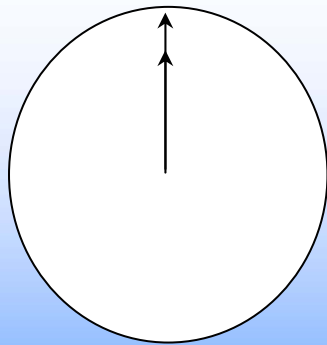**Quantity**/Public/Reflection/
Open-Constructed
Response

HK Mean % Correct: 52.75 %
OECD Mean % Correct: 40.34 %

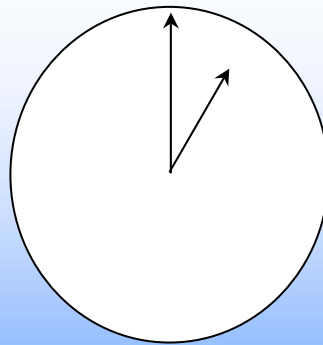# Sample Task 2—網上聊天 (Internet Relay Chat)

- 引文:

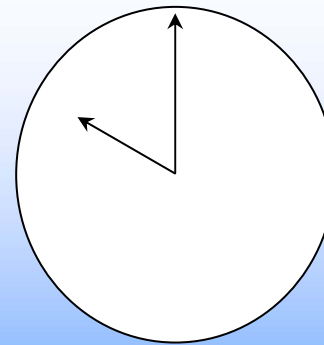 馬可（來自澳洲悉尼）及漢斯（來自德國柏林）常常以網上「聊天」的方式來溝通。他們需要同時登上互聯網才能聊天。

  為了選擇一個適合的時間聊天，馬可查看了世界各地相應的時間，並找得以下資料：

格林威治午夜 12時　　　　柏林 1:00 AM　　　　悉尼 10:00 AM

Sample Task 2—網上聊天 (Internet Relay Chat) (cont'd)

題目 1:
在悉尼的7:00 PM,柏林是什麼時間呢?

Change & Relationships/
Personal/Connections/Short Response

HK Mean % Correct: 47.24 %
OECD Mean % Correct: 53.72 %

題目 2:
由於需要上學的關係,馬可及漢斯不能於他們各自的當地時間9:00 AM至4:30 PM期間聊天。另外,由於他們各自的當地時間11:00 PM至7:00 AM是他們的睡眠時間,他們也不能聊天。
什麼時間才適合馬可和漢斯聊天呢?請在下表填寫當地時間。

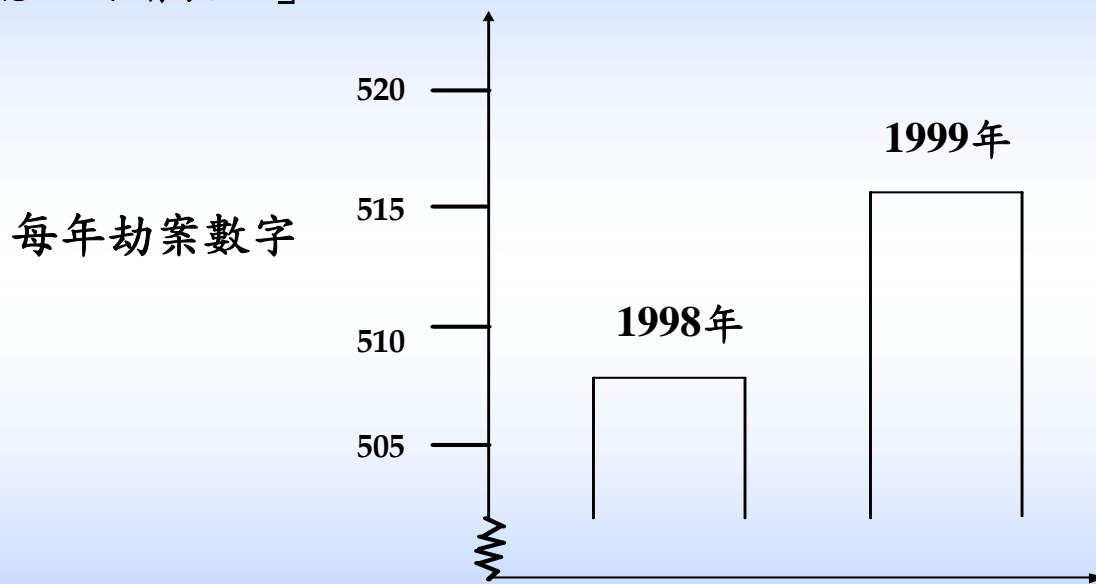| 地方	時間
悉尼	
柏林	

Change & Relationships/
Personal/Connections

HK Mean % Correct: 27.44 %
OECD Mean % Correct: 28.79 %

Sample Task 3—劫案(Robberies)

- 引文:

一名電視記者展示以下圖表,並說:「圖表顯示,1998至1999年間的劫案數字有龐大的增長。」

每年劫案數字

題目:
你認為這名記者對於這個圖表的詮釋合理嗎?
請提供一個解釋以支持你的答案。

Uncertainty/Personal/Connections/
Open-Constructed Response

HK Mean % Correct: 39.7 %
OECD Mean % Correct: 29.5 %

Sample Task 4—木匠 (Carpenter)

- 引文：
  木匠想用一條32米長的木條來圍著花圃。他正考慮用下列其中一個花圃的設計。
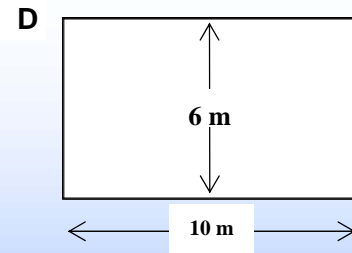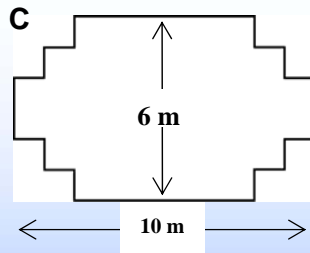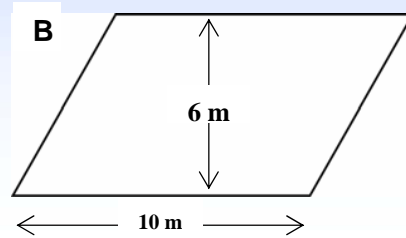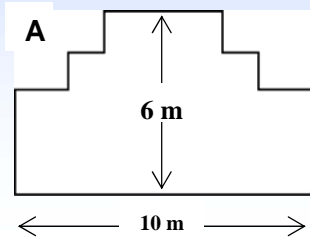
題目：
以上的花圃可以用32米長的木條造出來嗎？請在下表圈出「可以」或「不可以」。

花圃設計	若運用這設計，可以用32米木條造出來嗎？
設計 A	可以 / 不可以
設計 B	可以 / 不可以
設計 C	可以 / 不可以
設計 D	可以 / 不可以

Space & Shape/Educational/Connections/Complex Multiple Choice

HK Mean % Correct: 39.92 %
OECD Mean % Correct: 19.95 %

Mathematical Performance from HKPISA+ to HKPISA 2003

Mean Score and Percentile Comparisons between HKPISA+ and HKPISA 2003

	Mean Scores		Significance Level of Changes						
	HKPISA+	HKPISA 2003	5th	10th	25th	Mean	75th	90th	95th
Combined Mathematical Literacy	560	550	NA	NA	NA	NA	NA	NA	NA
Space & Shape	543	558	O	O	O	+	+++	+	O
Change & Relationships	546	540	O	-	O	O	O	O	O
Quantity	NA	545	NA	NA	NA	NA	NA	NA	NA
Uncertainty	NA	558	NA	NA	NA	NA	NA	NA	NA

Note: + and +++ represent "HKPISA 2003 higher than HKPISA+ at 90% and 99% confidence level respectively".
  - represents "HKPISA 2003 lower than HKPISA+ at 90% confidence level".
  O represents "no difference".

Since the coverage of mathematical assessment is extended to include two new subscales of "quantity" and "uncertainty" in PISA 2003, direct comparison is limited to the old subscales of "space and shape" and "change and relationships" between HKPISA+ and HKPISA 2003.

Overall Mathematical Performance in PISA 2003: Hong Kong VS Other Participating Countries

Combined Mathematical Scale

Country	Mean	S.E.
Hong Kong	**550**	**(4.5)**
Finland	544	(1.9)
Korea	542	(3.2)
Netherlands	538	(3.1)
Liechtenstein	536	(4.1)
Japan	534	(4.0)
Canada	532	(1.8)
Belgium	529	(2.3)
Macao	527	(2.9)
Switzerland	527	(3.4)

In combined mathematical scale:

• **HK achieves the top rank among all countries/regions with the mean score of 550 and a standard error of 4.5**

• **HK significantly outperforms all other countries/regions except Finland, Korea, Netherlands, Liechtenstein, and Japan.**

Overall Mathematical Performance in PISA 2003: Hong Kong VS Other Participating Countries (cont'd)

Space and Shape

Country	Mean	S.E.
Hong Kong	**558**	**(4.8)**
Japan	553	(4.3)
Korea	552	(3.8)
Switzerland	540	(3.5)

Change and Relationships

Country	Mean	S.E.
Netherlands	551	(3.1)
Korea	548	(3.5)
Finland	543	(2.2)
Hong Kong	**540**	**(4.7)**

Quantity

Country	Mean	S.E.
Finland	549	(1.8)
Hong Kong	**545**	**(4.2)**
Korea	537	(3.0)
Liechtenstein	534	(4.1)

Uncertainty

Country	Mean	S.E.
Hong Kong	**558**	**(4.6)**
Netherlands	549	(3.0)
Finland	545	(2.1)
Canada	542	(1.8)

In the four subscales:
- HK ranks 1st in both "space and shape" and "uncertainty", 2nd in "quantity", and 4th in "change and relationships" among all countries/regions.

Overall Mathematical Performance in PISA 2003: Hong Kong VS Other Participating Countries (cont'd)

Percentiles	HK	OECD
5th	374	332
10th	417	369
25th	485	432
75th	622	571
90th	672	628
95th	700	660
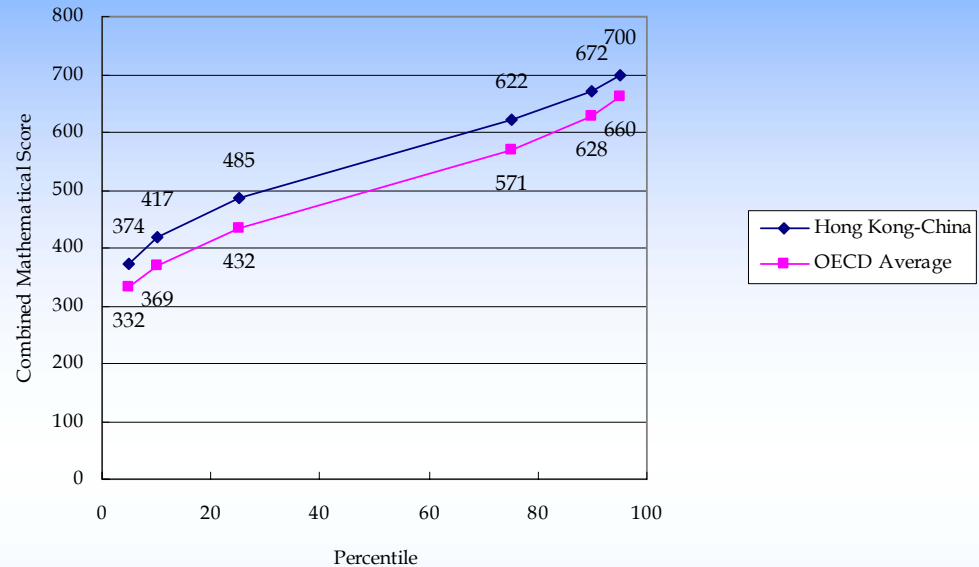
In percentile comparison:
• **The average score of HK students is always higher than the OECD average at each percentile, indicating that both lower and higher achievers in HK are doing equally well compared to the international counterparts.**

Overall Mathematical Performance in PISA 2003: Hong Kong VS Other Participating Countries (cont'd)

	HK Average (%)	OECD Average (%)
Processes		
Reproduction	76	66
Connections	57	46
Reflection	46	35
Situations		
Educational	62	47
Intra-mathematical	19	14
Occupational	62	57
Personal	63	54
Public	62	49
Scientific	55	46

In these two dimensions:
• HK average is greater than the OECD average in all categories.

To sum up:
• HK students have shown a remarkable mathematical performance when compared with other participating countries/regions on various dimensions of the PISA mathematical framework.

Disparity between High and Low Achievers

	Mean		Percentiles				Diff.
			5th		95th		
	Score	S.E.	Score	S.E.	Score	S.E.	
OECD Average	500	(0.6)	332	(1.3)	660	(1.0)	328
HKPISA 2003	550	(4.5)	374	(11.0)	700	(4.0)	326
HKPISA+	560	(3.3)	390	(10.3)	699	(5.0)	309
Finland	544	(1.9)	406	(3.8)	680	(3.1)	274
Korea	542	(3.2)	388	(4.6)	690	(6.8)	302
Japan	534	(4.0)	361	(8.2)	690	(6.6)	329

The disparity increases by 17 scores from the spread of 309 in HKPISA+ to 326 in HKPISA 2003.

# Distribution across Mathematical Proficiency Levels
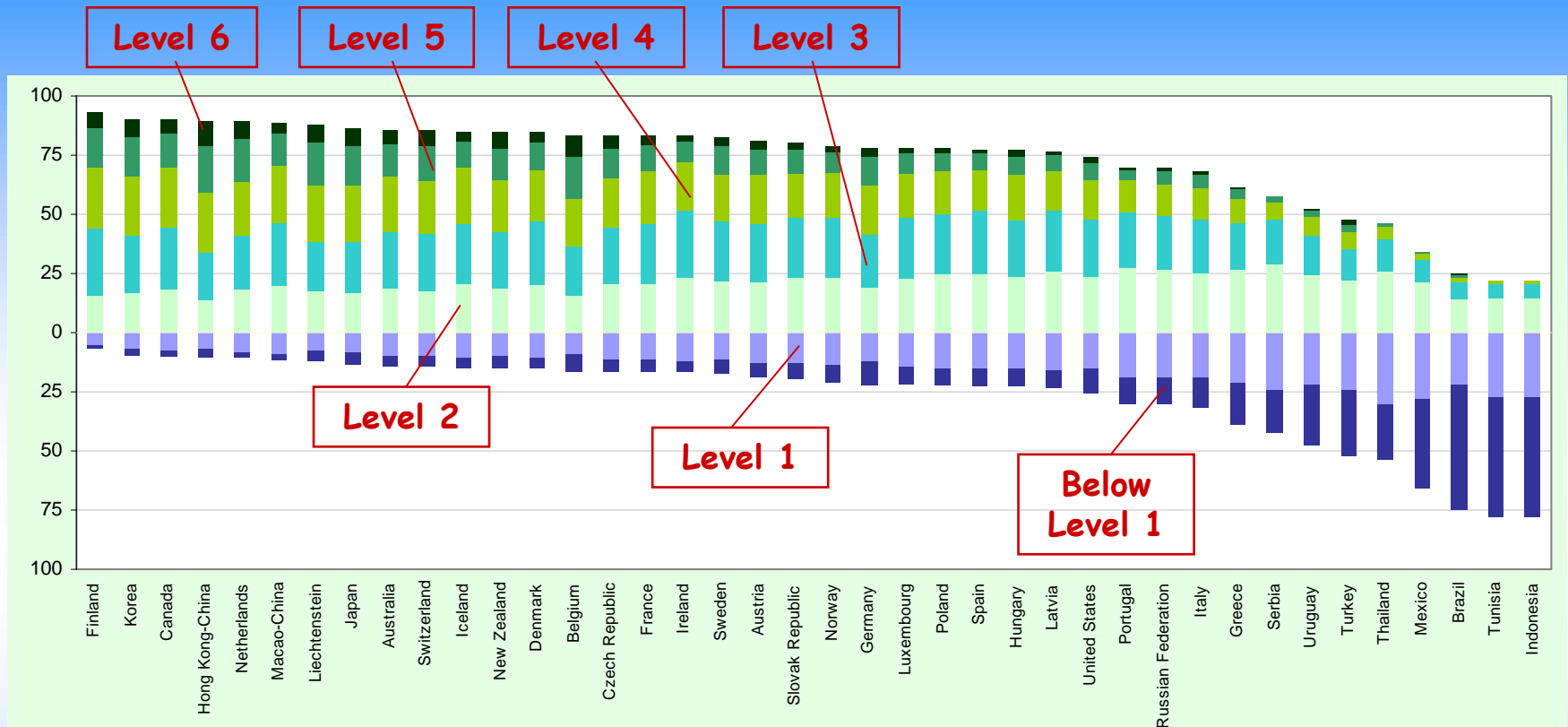
By using Level 2 as cut-off point, HK ranks 4th among all participating countries/regions with a total of 89.6% of students at Level 2 or above, only after Finland (93.2%), Korea (90.5%), and Canada (89.9%).

The rationale for using Level 2 as cut-off point is:
- Level 2 requires students to have sufficient direct reasoning and literal interpretations in mathematics.
- It then represents the basic ability of students to recognize, apply, and interpret simple given mathematical models.

Distribution across Mathematical Proficiency Levels (cont'd)

Percentage of HK Students at Each Mathematical Proficiency Level
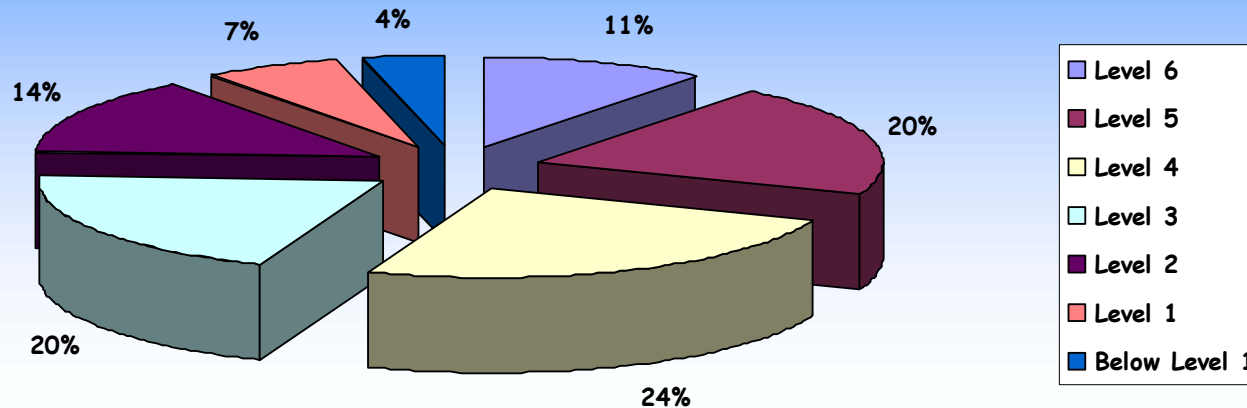
At the upper end:
- **Level 6 (HK-11%, OECD-4%)**
- **Level 5 (HK-20%, OECD-11%)**
- **HK total percentage (31%) as the 1st rank**

At the lower end:
- **Level 1 (HK-7%, OECD-13%)**
- **Below Level 1 (HK-4%, OECD-8%)**
- **HK total percentage (11%) as the 4th lowest rank**

To sum up:
- **HK highest mathematical performance is mainly due to the large % of students achieving higher levels of Level 4 to Level 6 (55%) as well as the relatively small % of students at lower levels of Below Level 1 and Level 1 (11%).**
- **HK has larger amount of higher achievers and smaller amount of lower achievers.**

Distribution across Mathematical Proficiency Levels (cont'd)

Proportion of Students at Each Proficiency Level of the Mathematical Sub-Scales

Levels	Space and Shape			Change and Relationships			Quantity			Uncertainty		
	HK	OECD	Diff.	HK	OECD	Diff.	HK	OECD	Diff.	HK	OECD	Diff.
Below 1	4%	11%	-7%	6%	10%	-4%	4%	9%	-5%	3%	7%	-4%
1	7%	14%	-7%	8%	13%	-5%	7%	13%	-6%	6%	13%	-7%
2	13%	20%	-7%	15%	20%	-5%	14%	20%	-6%	13%	22%	-9%
3	19%	22%	-3%	21%	22%	-1%	22%	24%	-2%	19%	24%	-5%
4	22%	17%	5%	23%	19%	4%	26%	20%	6%	25%	19%	6%
5	20%	10%	10%	19%	11%	8%	19%	11%	8%	21%	11%	10%
6	16%	6%	10%	10%	5%	5%	9%	4%	5%	13%	4%	9%

Note: There may be rounding errors.

For **lower levels** (Below Level 1 to Level 2): The % of HK students at these levels are lower than the OECD averages on all four subscales.

For **higher levels** (Level 5 to Level 6): The % of HK students at these levels are higher than the OECD averages on all four subscales.

For **medium levels** (Level 3 to Level 4): The % of HK students are lower and higher than the OECD averages at Level 3 and Level 4 respectively on all four subscales.

To sum up:
HK has **higher proportion of high achievers** and **lower proportion of low achievers** in terms of students' mathematical proficiency levels.

Summary

- **High quality in mathematical performance in the combined scale as well as the four subscales.**

- **Both lower and higher achievers are better than the international counterparts. And the disparity between them is similar to the OECD average.**

- **HK has more proficient students in mathematics.**

- **Low gender gap in mathematical learning and performance.**

Implications: Hong Kong Mathematics Curriculum Reform and Classroom Teaching

- **Emphasis of mathematics curriculum on equipping students with mathematical knowledge for future studies, workplace or daily life**

- **The relationship between real life and mathematical knowledge & skills**

- **Making sense of our daily life experience**

- **Relevancy to both teaching and learning in terms of assessment**

Thank You!!!