DEPARTMENT OF PHYSICS, CHINESE UNIVERSITY OF HONG KONG
PHYS3021 QUANTUM MECHANICS I

SAMPLE QUESTIONS FOR WEEK 6 EXERCISE CLASSES (9-13 October 2017)

Read me: TA will discuss the **SAMPLE QUESTIONS** (SQs) and answer your questions in exercise classes every week. The SQs are designed to review what you have learnt, tell a physics story, enrich our discussions, and help you work out the upcoming Problem Set. **Your time table should allow you to attend one exercise class session. The Exercise Classes are an integrated part of the course.** You are encouraged to work out (or think about) the SQs before attending exercise class and ask the TA questions. Over the semester, you are welcome to seek help from TAs/me.

Week 5 was interrupted by two public holidays
SQ14 - Think classical and Go quantum - Operators of quantities
SQ15 - The convenience of Lagrangian
SQ13 - Alternative Go Quantum method and TISE of oscillator in momentum space

SQ14 *Think classical and Go quantum - Operators of quantities*

With a way of expressing $\hat{x}$ and $\hat{p}$ for which the commutator is $[\hat{x}, \hat{p}] = i\hbar$, and knowing x and p are the variables to express the Hamiltonian H (classical mechanics), other quantities can also be expressed in terms of x and p and then turned into operators. Hence, QM operators of other quantities can be readily found.

We considered the components of the angular momentum in class notes. Thinking classically, it is $\mathbf{L} = \mathbf{r} \times \mathbf{p}$. Going quantum, $\hat{\mathbf{L}} = \hat{\mathbf{r}} \times \hat{\mathbf{p}}$.

TA: **Construct** the operator for the angular momentum squared $\hat{L}^2$. [Hint: Easiest way is to express it in terms of the components.] Then **demonstrate the steps** in evaluating the commutator $[\hat{L}^2, \hat{L}_x]$. [Students: This is related to what you did in Problem 2.6.]

[Remark: Note that the commutation relation depends **only** on the commutation relation $[\hat{x}, \hat{p}] = i\hbar$, but **only explicitly** on how to express the operators $\hat{x}$ and $\hat{p}$. For example, one may express operators has x, y, z and their derivatives, or (r, θ, ϕ) and their derivatives, or even go to the momentum space. What is the momentum space? See SQ16.]

SQ15 *The convenience of the Lagrangian*

The Hamiltonian, which has its origin from Hamilton's mechanics, is the starting point of doing Quantum Mechanics. The **formal way** (not necessarily the practical way though) in writing down the Hamiltonian follows that in classical mechanics, namely starting with the Lagrangian → finding conjugate momentum for each coordinate → transforming to Hamiltonian. **Practically**, we often start by expressing the Hamiltonian as the sum of kinetic energy and potential energy terms.

(a) For a particle m moving in a 2D plane, its position $\mathbf{r}$ can be expressed as (x, y) or in plane polar coordinates $(r\cos\theta, r\sin\theta)$. Similarly, one could write down $\dot{\mathbf{r}}$, etc. With the kinetic energy terms (2D) only, the Lagrangian can be expressed as

$$L = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) \tag{1}$$

Note that we don't include a potential energy function $V(x, y)$ or $V(r, \theta)$ into the consideration here. There are two coordinates r and θ.

TA: **Identify** the momentum p_r associated with r and the momentum p_θ associated with θ. Hence, **write down** the equations of motion using the Euler-Lagrange equation. **Inspect** $L(r, \dot{r}, \theta, \dot{\theta})$ and **identify** which one is a conserved momentum. **Construct** the Hamiltonian.

(b) **2D rigid rotator (or rotor).** Next, we consider a special case in which the particle is restricted to move only on a circle of radius R, i.e. $r = R$ being a constant. **(Think classical)** In this case, θ is the only coordinate. TA: **Identify** the momentum and **construct** the Hamiltonian. **(Go Quantum)** Turn θ and p_θ into operators and **write down** the time-independent Schrödinger equation. You may want to define the moment of inertia I in expression the resulting equation.

SQ16 *1D oscillator in momentum space*

According to Dirac (1925 paper, see class notes), the commutator $[\hat{x}, \hat{p}] = i\hbar$ is **all** of quantum mechanics. We showed in class that the substitution $\hat{x} \to x$ and $\hat{p} \to \frac{\hbar}{i}\frac{d}{dx}$ indeed satisfies the relation.

TA: There may be other ways to do the job. TA: **Check** that the substitution $\hat{x} \to i\hbar\frac{d}{dp}$ and $\hat{p} \to p$ also satisfies Dirac's relation. In this way, operators act on functions of the momentum p, i.e. $f(p)$. We are thus working in the momentum space.

TA: Starting with the Hamiltonian (think classical) of an oscillator

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 \ , \tag{2}$$

go quantum by making the substitution to **obtain** TISE of an oscillator in the momentum space.

In Problem 2.5, students worked on the 1D oscillator problem in position space (or x-space) and tested a given wavefunction to be an energy eigenfunction with the lowest energy (ground state wavefunction) and normalized it. There, everything was written in x and the wavefunction is $\psi_0(x)$.

TA: **Copy down** the corresponding TISE in position space and the corresponding ground state wavefunction (from solutions to Problem 2.5). By comparing the forms of TISE's in momentum space and coordinate space, **suggest** a form of the ground state wavefunction $\phi_0(p)$ in the momentum space. [No detailed calculations needed here. Just draw analogy between the two equations and write down the wavefunction.]

Hence, **test** that your proposed $\phi_0(p)$ is really a solution to TISE in momentum space (thus $\hat{H}\phi_0(p) = E_0\phi_0(p)$) and **find** the corresponding eigenvalue (which is the ground state energy).

[Remarks: This illustrates that one can do QM not only in the x-space, but also in the p-space. The results contain no more and no less information. We usually work things out in x space because we have a better sense (or we think that we have a better sense) when considering the physics in x-space. As the physics content is the same, there is no reason to train yourself intentionally to do physics in the momentum space, until you become very familiar with the standard treatments. **Students**: In Problem 2.5, you started in the position space and did a Fourier transform to go into the momentum space. It will be an interesting exercise to re-phrase Problem 2.5 by yourself and re-do it by starting with the momentum-space and then doing a Fourier transform to go (back) to the (more familiar) position-space.]