DEPARTMENT OF PHYSICS, CHINESE UNIVERSITY OF HONG KONG
PHYS3021 QUANTUM MECHANICS I

SAMPLE QUESTIONS FOR WEEK 4 EXERCISE CLASSES (25-29 September 2017)

Read me: TA will discuss the **SAMPLE QUESTIONS** (SQs) and answer your questions in exercise classes every week. The SQs are designed to review what you have learnt, tell a physics story, enrich our discussions, and help you work out the upcoming Problem Set. **Your time table should allow you to attend one exercise class session. The Exercise Classes are an integrated part of the course.** You are encouraged to work out (or think about) the SQs before attending exercise class and ask the TA questions. Over the semester, you are welcome to seek help from TAs/me.

SQ11 - Mathematical operations and operators
SQ12 - TISE is an eigenvalue problem - Be patience in checking a function for solution
SQ13 - Normalization, evaluating mean and variance of position

SQ11 *Mathematical operations or operators*

(a) Many students are confused about what linear terms are about in an equation. Let's represent the consecutive operations for the same operators, e.g. $\hat{A}\hat{A}$ as $\hat{A}^2$. Consider $\hat{A} = \frac{\hbar}{i}\frac{d}{dx}$. **Test** that $\hat{A}^2 f(x)$ and $[\hat{A}f(x)]^2$ are two different things and they give different results. In addition, **show** that $\hat{A}^2$ is linear and the other one is nonlinear.

(b) Take $\hat{B} = x^2$, i.e., take in whatever $f(x)$ comes in on the right and multiply it by x^2. Using $\hat{A}$ and $\hat{B}$ in parts (a) and (b), **work out** what $\hat{A}\hat{B}f(x)$ and $\hat{B}\hat{A}f(x)$ are for any $f(x)$. Hence, **construct** the comutator $[\hat{A}, \hat{B}]$.

SQ12 *TISE is an eigenvalue problem - Be patience in checking a function for solution* (See Problem Set 2)

In PHYS3021, we will soon solve the time-independent Schrödinger equation for various problems. Schrödinger (1926) showed that the eigenvalues E turned out to be the allowed energies of a system, all came out from solving his one equation.

Before we solve TISE, an easier task is to check whether a given wavefunction is a solution or not. Consider a mass m in one-dimension (1D) under the influence of a restoring force and thus a parabolic potential energy function of the form $U(x) = \frac{1}{2}m\omega^2 x^2$.

(a) **Write down** the time-dependent Schrödinger equation. [Don't need to invoke operators. Just write it down.]

(b) **Write down** the time-independent Schrödinger equation.

(c) A student, after some magical thought, suggested a function of the form

$$\psi(x) = A^{1/2}\, x\, \exp\left(-\frac{m\omega x^2}{2\hbar}\right) \tag{1}$$

to be a possible solution to TISE. **Check explicitly** if the claim is correct or not. If yes, **find** the energy E corresponding to this wavefunction. [Note: Since A is meant to be a factor that does not depend on x, we need not determine A before testing $\psi(x)$ on TISE.]
[Remark to Students – Try this SQ out yourself. If you can get the correct answer in one shot, congratulations! This is an example that shows doing derivatives patiently is an important skill in QM. If you think that "I have known differentiations for 6 years (but not trying it out)", it is the most dangerous. Practice, practice, practice!]

SQ13 *Continuing on SQ12 - 1D harmonic oscillator* (See Problem Set 2)

Continuing on SQ12 and using Eq. (1)...

(a) **Normalize** the wavefunction $\psi(x)$ in Eq. (1).

(b) **Sketch** $\psi(x)$ and $|\psi(x)|^2$.

(c) **Evaluate** the mean position $\langle x \rangle$ and the variance $(\Delta x)^2$ via

$$\langle x \rangle = \int_{-\infty}^{\infty} x|\psi(x)|^2 dx = \int_{-\infty}^{\infty} \psi(x)^* \, x \, \psi(x) dx \tag{2}$$

$$(\Delta x)^2 = \int_{-\infty}^{\infty} (x - \langle x \rangle)^2 |\psi(x)|^2 dx = \langle x^2 \rangle - \langle x \rangle^2 \tag{3}$$

Note that Δx is formally the uncertainty in position in QM.

(d) **Discuss** the problem when the center of the restoring force is not at $x = 0$ as in SQ12 and SQ13(a)-(c) but shifted to $x = x_0$. Should the physics be the same (what does it mean)?