DEPARTMENT OF PHYSICS, CHINESE UNIVERSITY OF HONG KONG
PHYS3021 QUANTUM MECHANICS I

SAMPLE QUESTIONS FOR WEEK 2 EXERCISE CLASSES (11-15 September 2017)

Read me: TA will discuss the **SAMPLE QUESTIONS** (SQs) and answer your questions in exercise classes every week. The SQs are designed to review what you have learnt, tell a physics story, enrich our discussions, and help you work out the upcoming Problem Set. **Your time table should allow you to attend one exercise class session. The Exercise Classes are an integrated part of the course.** You are encouraged to work out (or think about) the SQs before attending exercise class and ask the TA questions. Over the semester, you are welcome to seek help from TAs/me.

SQ5 - Young's two-slit experiment with light
SQ6 - Young's two-slit experiment with atoms (a classic paper)
SQ7 - What do wave equations do?

SQ5 *Young's two-slit experiment with Light*

Background: In 2002, the Institute of Physics (IOP) in UK held a poll on *the most beautiful experiments in physics*. Ranked #1 was "the Young's double-slit experiment applied to the interference of single electrons" and Ranked #5 was "Young's light-interference experiment". [Google "the most beautiful experiment" and see items under physicsworld.com.] Chapter I focused on these two experiments (for light and dim source AND for electrons and dim source).

This traditional question on Young's two-slit experiment using light aims to point out that the experiment remains useful in determining the wavelength. Consider a standard double-slit set up. The slits are $d = 0.40$ mm apart. The slits and the plane of observation (the screen full of detectors or a detector moving around) are $L = 600$ mm apart.

[Hint to students: The key formula in two-slit experiments is that constructive interference (and thus intensity maxima) occurs when $d \sin\theta = m\lambda$, with symbols that you need to find that what they are.]

Often it is difficult to measure accurately the separation between neighboring maxima (bright fringes) on the screen as the separation may be small. Let's say someone measured that the separation between the m-th maximum and the $(m + 10)$-th maximum on the screen is 7.5 mm. TA: **Find** the wavelength of the light being used. [Remark: This is how Young's experiment (or the extension to many-slits (grating)) can be used to determine the wavelength, without knowing what is waving. When electrons are used, the experiment can give the matter wavelength (de Broglie wavelength).]

In using Young's experiment *as a tool*, one can tune some parameters in the set up. Here we only change one number in each consideration. Referring to the same set up,

(a) if the separation of the two slits is altered to $d = 0.20$ mm, what will be the separation between the m-th maximum and the $(m+10)$-th maximum on the screen for the wavelength obtained above?

(b) if we keep d but put the screen further back from the slits to $L = 800$ mm, what will be the separation between the m-th maximum and the $(m + 10)$-th maximum on the screen?

(c) if we don't change d and L, but use another light source of wavelength $\lambda = 700$ nm, what will be the separation between the m-th maximum and the $(m + 10)$-th maximum on the screen?

SQ6 *Young's experiment using atoms - a classic paper in PRL 1991*

Background: After many successful experiments using electrons that clearly showed particle-wave duality, physicists started to use heavier objects in two-slit experiments. Up to now, atoms, C_{60}

molecules and even big dye molecules (500 atomic mass unit) have been used and passed the (quantum) test. This SQ is based on a modest but classic attempt using helium atoms by Carnal and Mlynek on "Young's double-slit experiment with atoms: A simple atom interferometer", Physical Review Letters 66 (1991) 2689. See https://journals.aps.org/prl/pdf/10.1103/PhysRevLett.66.2689 (Click here for soft copy)

I encourage students (and TAs) to browse through the paper to get a sense of how difficult such experiments are. Not mentioning the details, basically the two slits are separated by $d = 8\ \mu m$. The slits and the plane of observation (the screen) are separated by $L = 64\ cm$. The authors relied heavily on the observed periodic brighter and darker fringes and said that the periodicity (the period from one maximum to the next) is given by $L\lambda_{dB}/d$, where λ_{dB} is the de Broglie (matter) wavelength of the helium atoms. TA: **Show** how their equation comes out from the standard equation in two-slit set up.

In one experiment, the distance between two maxima on the screen was found to be $7.7\ \mu m$. **What** is the corresponding λ_{dB}? In another experiment, the distance between two maxima was found to be $4.5\ \mu m$. **What** is the corresponding λ_{dB}?

The authors controlled the velocity (and thus the momentum) of helium atoms by heating (thus by the temperature). At a temperature, there will be a mean velocity (Maxwell distribution of speeds in a gas). The authors used $T = 295\ K$ in one experiment and $T = 83\ K$ in another. TA: **Estimate** λ_{dB} at these two temperatures and **compare** the numbers that those obtained by two-slit experiments?

SQ7 *What do wave equations do? ("Examples from Classical Physics")*

Background: We will do many calculations using the wave equation of quantum mechanics, namely the *time-dependent Schrödinger equation* and the *time-independent Schrödinger equation.* This SQ wants to point out, whatever the waves may be, what wave equations do. In essence, a **wave equation** governs how a wave propagates (in time) in a physical system. In more detail, the wave equation gives the wave form, frequency (related to time), and wavelength (related to space) of each of the normal modes (meaning there are many normal modes). The propagation problem is then solved by decomposition a given initial situation (e.g. pluck a guitar string) into normal mode components, let each component propagate according to its own time factor, and then put the components back at the time you want to know the answer (as discussed in class).

Here, we focus on the point that the wave equation relates the frequency and the wavelength (thus wavenumber $k = 2\pi/\lambda$). Such a relation is called the **dispersion relation**.

(a) Look up the EM wave equation in vacuum and use a standard form of $\sim \cos(kx - \omega t)$, **show** that the expected relation for EM waves in vacuum comes out.

(b) The wave equation for a (guitar) string is

$$\frac{\partial^2 \psi(x,t)}{\partial x^2} = \frac{\mu}{T}\frac{\partial^2 \psi(x,t)}{\partial t^2} \tag{1}$$

where T is the tension (you tune the guitar by tightening the screw because that will alter the tension) and μ is the mass per unit length of the string (that's why the six strings in a guitar are different), and $\psi(x,t)$ is the displacement of the string from equilibrium (not vibrating). Use the same standard wave form to related ω and k. Now, your guitar string is of a certain length (fixed at two ends) L. The fundamental (the normal mode of the longest wavelength) has a wavelength fixed by L. Hence, a string (free string, not pressed down somewhere) has a characteristic frequency (e.g. the first and the sixth strings are E but differ by 2 octaves). **Express** the frequency of the fundamental in terms of L, T, and μ.