DEPARTMENT OF PHYSICS, CHINESE UNIVERSITY OF HONG KONG
PHYS3021 QUANTUM MECHANICS I

SAMPLE QUESTIONS FOR WEEK 1 EXERCISE CLASSES (4-8 September 2017)

Read me: What are Sample Questions (SQs)?

TA will discuss the **SAMPLE QUESTIONS** (SQs) and answer your questions in exercise classes every week. The SQs are designed to serve several purposes. They review what you have learnt in previous courses that are needed here, tell a physics story, enrich the discussions in lectures, and some are *closed related* to the questions in an upcoming Problem Set. Students should be able to do the homework problems independently after attending the exercise class. **Your time table should allow you to attend one exercise class session. The Exercise Classes are an integrated part of the course.** You are encouraged to work out (or at least think about) the sample questions before attending exercise class and ask the TA questions. Over the semester, you are welcome to seek help from TAs/me.

SQ1 - How big is an atom (an estimate from macroscopic quantities)?
SQ2 - Useful numbers to carry along (electrons in quantum physics)
SQ3 - Properties of light used in Compton scattering
SQ4 - Changing variables in distributions (continuous variables)

SQ1 *How big is an atom?*

In 1920's, Quantum Mechanics was formulated to solve the puzzles related to the physics of atoms. In particular, each atom has its characteristic spectrum, signifying that the allowed energies of the whole atoms can only take on some discrete values. Schrödinger's first paper (1926) and Heisenberg's first paper (1925) were targeted at explaining the hydrogen spectrum. Prior to these works, there was Bohr's model (1913) of hydrogen. The spectrum problem was already too difficult. The size of an atom was a more fundamental question, i.e. why are atoms always of that range of sizes? One of the contributions of Bohr is to provide an answer using his Bohr's model to provide an answer (the size is about that of the Bohr's radius).

TA: Here, we get a sense of the size of an atom by an estimation. Given that diamond has a density of 3.5×10^3 kg m^{-3} and carbon has a relative atomic mass of 12.0, **estimate** the radius of a carbon atom. **Give** the answer in units of m and Å.

[Remarks: (i) "Making estimation" is an important skill. In doing so, just make whatever sensible approximations and try to get at an answer. (ii) As we get into quantum physics, we will see how the "size" comes about. We could use the uncertainty relation and the variational method to get approximations, and solving the Schrödinger equation for hydrogen exactly to study the problem. Stay tuned.]

SQ2 *Useful numbers to carry along (Electrons in Quantum Physics)*

In physics, there is a number that characterizes a subject. In EM (and special relativity), we have the speed of light c. In statistical physics, we have the Boltzmann constant k_B. In quantum physics, we have the Planck's constant h or $\hbar = h/2\pi$.

(a) **State** the values of h and $\hbar$. [Remarks: In recent years, laborious experiments have been done to measure h as accurately as possible, in an effort to re-define the kilogram.]

(b) In considering atoms, we are concerned with **electrons** under the influence of the nucleus's attraction. So **numbers related to an electron in quantum formats** are particularly important.

  (i) **Work out** the combination $\hbar^2/m_e$ in units involving eV and Å, where m_e is the electron mass.

(ii) **Work out** the combination $e^2/(4\pi\epsilon_0)$ in units involving eV and Å.

(c) Using the estimated size (radius) of an atom in SQ1 and the answer in b(i), **estimate** the energy of an electron confined to an atom. [Remark: If you provide this much energy, the electron can be set free from the atom. Thus, it is an estimation of the ionization energy of an atom.]

[Remarks: Take these numbers $\hbar^2/m_e$ and $e^2/(4\pi\epsilon_0)$ with you. It will make you look "pro" in doing atomic physics and solid state physics. A twist on the question is to turn the numbers into the context of nuclear physics.]

SQ3 *Particle Nature of Light - Photons*

The standard discussion would be: Photoelectric effect establishes $E = hf$ or $E = \hbar\omega$ as the energy of photons for an EM waves of frequency f. The Compton scattering experiment further established that a photon has a momentum $p = h/\lambda = \hbar k$. In the Compton effect, the photon is treated as a particle that collides with an electron and it loses some energy so that its wavelength becomes longer. It is often said that in the Compton experiment (and many experiments that demonstrated the particle nature of light and the wave nature of particles), "the wave nature of light was also used in addition to the particle nature of light".

TA: **Discuss** this statement without doing any derivation of the Compton effect.

SQ4 *Planck's Formulas for Thermal Radiation*

In PHYS1122 (see notes), you saw the Planck's formula for thermal radiation. There are different forms, depending on the exact quantity in question. One quantity is the energy per unit volume in the range of wavelengths between λ and $\lambda + d\lambda$ at a temperature T given by

$$u(\lambda, T)d\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{\exp(\frac{hc}{\lambda k_B T}) - 1}\,d\lambda. \tag{1}$$

[If we plot this out, the x-axis will be the wavelength. See class notes for a plot.]

The Planck's formula can also be expressed in terms of the frequency f. It looks easy because $c = f\lambda$ and a change of variable will do the job. The end result is

$$\tilde{u}(f, T)df = \frac{8\pi f^2}{c^3}\frac{hf}{\exp(\frac{hf}{k_B T}) - 1}\,df. \tag{2}$$

TA: **Show** the math carefully.