Quantum Hall Effect

What so special about Quantum Hall Effect??
What is the difference between Classical and Quantum?

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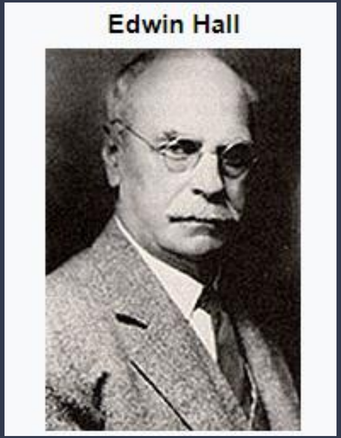
Background



Credit: Wikipedia

Classical Hall Effect (Some Recap)

- Discovered by Edwin Hall in 1879
- Occurs when we apply a B-field perpendicular to the current (x-y plane)
- The electrons will experience a Lorentz force and being pushed to the sides
- Creating a potential difference across

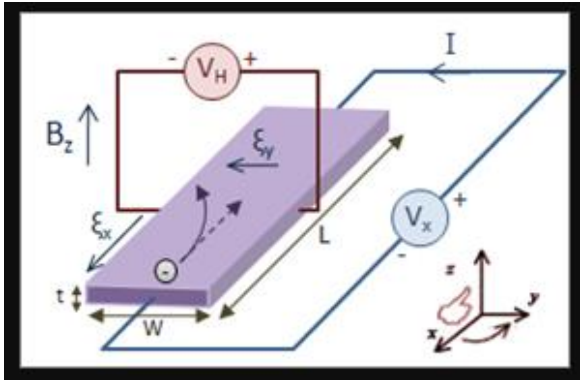


Fig.1: Schematic Diagram for Classical Hall Effect

Classical Hall Effect

- When we apply a B-field in the z-direction, perpendicularly to a 2-D material
- The Lorentz force is, using the Drude model:

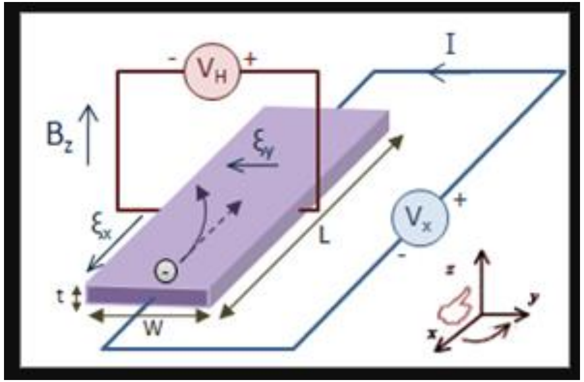
$$\vec{F} = m\frac{d\vec{v}}{dt} = -|q|\vec{E} - |q|\vec{v}\times\vec{B} - \frac{m\vec{v}}{\tau}$$; τ is the average collision time between electron and ion

We know: $\vec{v} = \frac{\vec{J}}{n_e|q|}$

- At equilibrium, we have:

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \frac{1}{\sigma_D}\begin{bmatrix} 1 & \omega_B\tau \\ -\omega_B\tau & 1 \end{bmatrix}\begin{bmatrix} J_x \\ J_y \end{bmatrix}$$

$$\sigma_D = \frac{|q|^2 n_e\tau}{m}, \omega_B = \frac{|q|B\tau}{m}$$



Classical Hall Effect (cont.)

$$\rho_{xx} = \frac{1}{\sigma_D} = \frac{m}{|q|^2 n\tau}$$

$$\rho_{xy} = \frac{\omega_B \tau}{\sigma_D} = \frac{B}{|q|n}$$

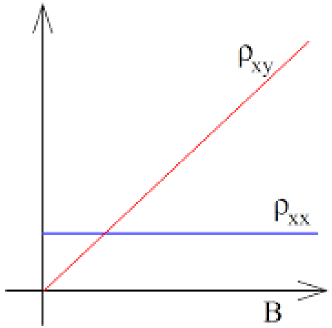


Fig.2: Resistivity against B-field

- We can see ρ_{xx} (**Longitudinal resistivity**) and ρ_{xy} (**Hall resistivity**) are constant and proportional to B respectively

The leap: From classical to quantum

- **Klaus von Klitzing** made an unexpected discovery in 1980 when he applied very strong B-field under extremely low temperature **(~4.2K)** on **2D electron gas**.
- What he expects: Resistance should be $\propto B$ (Classical)
- What he gets: The resistance depends only on **fundamental constants** and **integer numbers** only!!
- A Nobel prize is awarded (1985)

Credit: wiki/Klaus_von_Klitzing

The discovery of Quantum Hall Effect (QHE)

- Hall coefficient turns out to be quantized
- ρ_{xx} only exist where Hall coefficient is not constant
- Vanishing as much as 13 order of magnitude at the plateau
- For a filling factor $\upsilon = \frac{n_e h}{qB}$ (which we will address later), the Hall resistivity:

$$\rho_{xy} = \frac{h}{\upsilon q^2} \text{ where } \nu = 1,2,3 \ldots$$

- When $\nu = 1$, we have the **von Klitzing constant**, where

$$\boldsymbol{R_{k-90}} = 25812.807\ \Omega \text{ (exact)}$$

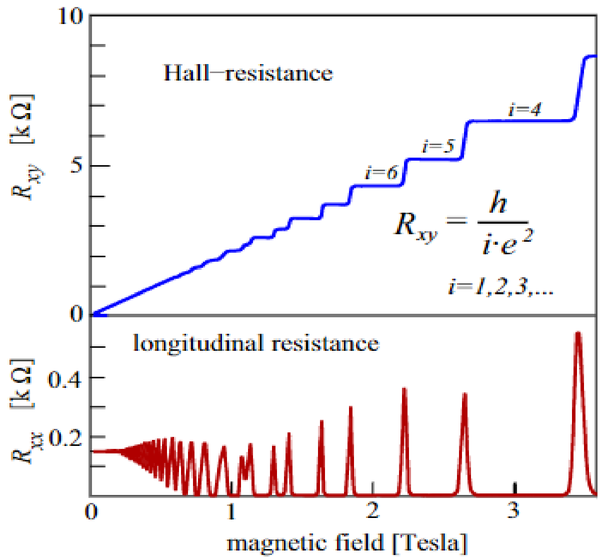


Fig.2: Resistance against B-field

The various QHEs

- Here we will provide a more quantum mechanical view on the various QHEs

2D Electron Gas

- 2D electron gas:
- Electrons can only move in 2D and being confined by some potential tightly in the third (infinite potential well)
- Hence, we can separate the z-direction eigenenergy out, which can be easily solved
- Therefore, the eigenenergies in the z-direction can be "ignored" in most problems

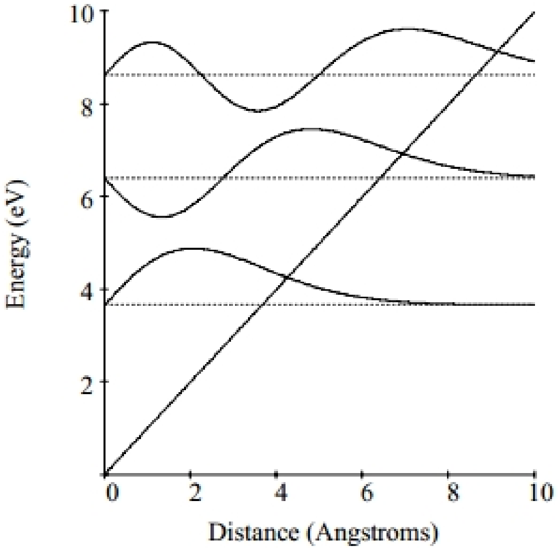


Fig.4: Triangular potential well

Adapted from http://transport.ece.illinois.edu

Integer QHE

- For a 2-D electron gas along the x-y plane and an external B-field applied in the z direction, the Schrödinger equation gives:

$$\hat{H}\psi(x,y) = \left[\frac{1}{2m}\left(\widehat{P_y} + exB\right)^2 + \frac{1}{2m}\left(\widehat{P_x}\right)^2\right]\psi(x,y)\ ;\ \widehat{P_{x/y}} = \frac{\hbar}{i}\frac{d^2}{d\left(\frac{x}{y}\right)^2}$$

- As the vector potential doesn't depend on y, we can simply assume a solution:

$$\psi(x,y) = e^{ik_y y} f(x)$$

i.e. the wave propagate freely in the y-direction

$$\hat{H}\psi(x,y) = e^{ik_y y}\left[\frac{\left(\hbar k_y + exB\right)^2}{2m} + \frac{1}{2m}\left(\widehat{P_x}\right)^2\right] f(x)$$

Integer QHE

$$= e^{ik_y y}\left[\frac{m\omega_B^2}{2}\left(x + k_y l_B^2\right)^2 + \frac{1}{2m}\left(\widehat{P}_x\right)^2\right] f(x)\ ;\ \omega_B = \frac{eB}{m}, l_B = \sqrt{\frac{\hbar}{eB}}$$

- Which is just a 1-D harmonic oscillator model shifted by $x_0 = k_y l_B^2$, thus we have eigenenergy:

$$E = \hbar\omega_B\left(n + \frac{1}{2}\right)\ ; n = 0,1,2,3\ ...$$ (This is our Landau Level)

- Note: As the energy does not depend on k_y, for a given n, it can have any k_y
- To consider degeneracy, we require:

$$e^{ik_y(0)} = e^{ik_y(L_y)} = 1$$

$$k_y = \frac{2\pi m}{L_y}\ ; m = 0,1,2\ ...$$

Integer QHE: Density of States

- How many k_y states are there?

$$x_0 = k_y l_B^2 = \frac{\hbar k_y}{eB}$$

- As x_0 must be within the sample, so

$$L_x \geq \frac{\hbar k_y}{eB} \rightarrow k_y \leq \frac{eBL_x}{\hbar}, \qquad k_{y,max} = \frac{2\pi m_{max}}{L_y} = \frac{eBL_x}{\hbar}$$

$$m_{max} = \frac{L_x L_y}{2\pi l_B^2}\ ; l_B = \sqrt{\frac{\hbar}{eB}}$$

- Hence, the no. of k_y states per unit area for a given n is $\frac{eB}{h}$; we get the filling factor $v = \frac{n_e h}{eB}$
- Assume B is strong enough such that s Landau Levels provide enough k_y states for all electron, then:

$$\rho_{xy} = \frac{B}{|q|n_e} = \frac{B}{|q|\left(s\frac{|q|B}{h}\right)} = \frac{h}{sq^2}$$

Fractional, Spin and Anomalous QHE

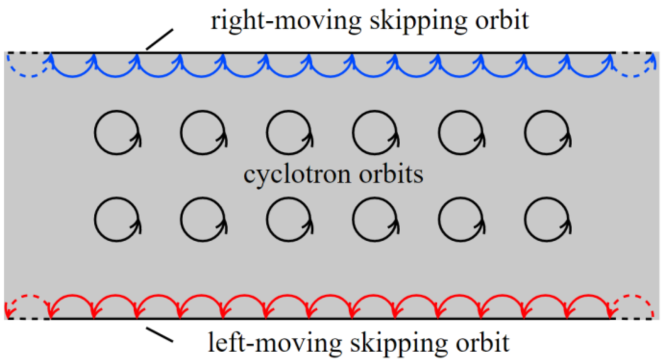
- **Fractional QHE**
  - When the B-field is further increased, electrons finally ends up in the lowest Landau level, the electron-electron interaction become dominant.
  - The Hall resistance now depends on some **fractional numbers.**

- **Spin QHE**
  - Due to the spin-orbit interaction of the electrons and no strong B-field is need to apply
  - Conductance = $\frac{2}{R_{K-90}}$

- **Anomalous QHE**
  - Quantum version of the classical anomalous hall effect, which exist in ferromagnetic material without B-field
  - Much greater contribution than conventional QHE
  - The integer of this QHE is equal to the Chern Number

Edge State

- It is easy to see whyd edge state exist, circular orbit for electron under perpendicular E and B field
- When orbit radius > length of sample → they bounce back
- Skipping orbits occur
- **Impurities will not affect edge state!!**
- The electrons are immune to backscattering — **topologically protected**
- There can be multiple edge states near the edge of the material
- Used to make conducting material with extremely high conductivity



Adapted from https://topocondmat.org/w3_pump_QHE/QHEedgestates.html

Deriving Hall Resistivity from Edge States

- At the edge state, the electrons are not allowed to reverse their velocity
- These states act as a transport channel with highest conductance, each with $\sigma_{max} = \frac{q^2}{h}$
- For a potential V, with n said channels, we get:

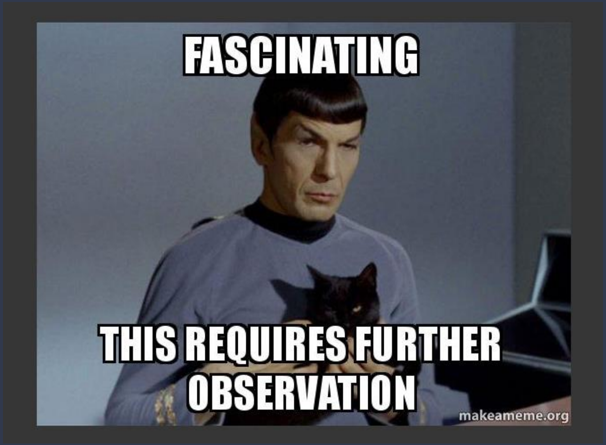
$$I_H = n\frac{q^2}{h}V$$

$$\sigma_H = n\frac{q^2}{h}$$

- Which agrees with our derivation before

Some Experimental observation

- Here we introduce some experimental results related to QHE

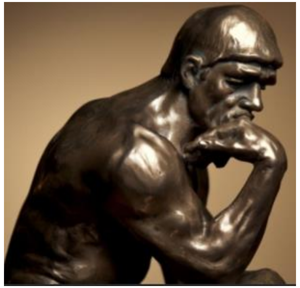


Credit: makeameme

Universality of QHE

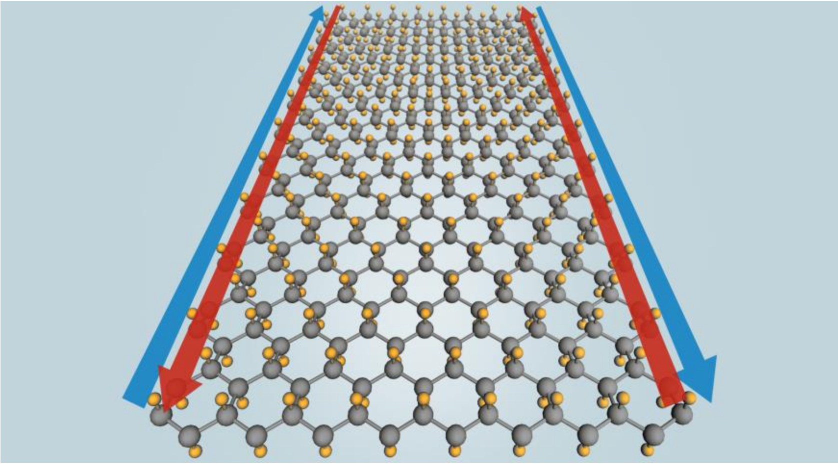
- In 1987, 4 different sets of **GaAs** were used to measure the Quantum Hall Resistance (QHR)
  - Their results agree in the range of 5×10^{-9}
- Further experiments were conducted with using **Si-MOSFET** and **GaAs** to probe the QHR
  - The difference was found to smaller than 3.5×10^{-10}
- Evidence that the **QHR is the same !**
  - Despite **any shape**, **material**, **fabrication process** or other factors
  - Showing his value is indeed fundamental
- Nowadays, the QHR is used by all the institute as a standard for resistance
- What can we further explore on this standardization??

What do this tell us??



Edge state related experiment: Stanene

- The “cousin’ of **graphene**, hexagonal structure but with **Tin (Sn)** as the atom
- A relative new type of material
  - First introduced in **2013**
- It conduct electricity **on the edge,** due to edge state,
- The **interior part** acts as an electrical insulator
- A topological insulator!



Adapted from: https://www.extremetech.com/

Stanene: what so special about it?

- Theoretically, Stanene is predicted to conduct electricity with **no dissipation** under **room temperature**
- The current will **not dissipate** even when there are **impurities** (Again due to properties of edge state)
  - unlike other materials such as silver or copper
  - A very special superconductor!
- **This year**, coexistence of these two properties **have been observed** on a few-layer Stanene
- Researchers grow Stanene successfully on the **Bi (Bismuth)** substrate
  - Stable in room temperature
  - Exhibit stable edge states and superconducting properties
- What can we further exploit from this Stanene??

Implication and outlook

Significance

- The **universality** of QHR provide us a way to redefine many units with fundamental constants, such as:
  - **Ω**, from the **von Klitzing constant**
  - **V**, by **Josephson effect**
  - **kg**, by using **Kibble balance setup**
  - you name it….

- Further improvement on the value of Fine-Structure constant $\boldsymbol{\alpha}$
  - An important measure on the strength of electromagnetic interaction
  - $\boldsymbol{\alpha^{-1} = \frac{2R_k}{\mu_0 c} = 137.036}$ ...with uncertainty = 10^{-9}!!
  - The more precise the better!

Significance

- Material revolution, Stanene for example
    - A superconductor in **room temperature**, low temperature is no longer a concern
    - **ZERO** energy dissipation - energy saving

- The "Quantumness" of nature
    - The QHE demonstrate evidence that our world is quantum in nature
    - Many units can be expressed by the **Planck constant *h***
    - Our quantum theory works well on solving these phenomenon

Branching with other fields of physics

- QHE is not only useful in solid state physics, but also useful in many other modern physics:
  - Particle Physics
  - Quantum Computation (Exploit FQHE to form topologically protected qubit)
  - Even String Theory (String Theory display FQHE)
  - More…

- When someone make some discovery on QHE, they often win a Nobel prize....
  - Klitzing, 1985 (Integer QHE)
  - Laughlin, Stormer and Tsui, 1998 (Fractional QHE)
  - And there is still Spin and Anomalous QHE left, **who is next** ??

Outlook

- Fully workable Stanene
    - Even workable in room temperature
    - Can be **massively** produced
    - Use it to build chips, other electronic devices, even quantum computers....

- Understand more on quantum entanglement & information processing
    - As QHE gives a basic model on quasiparticles such as composite-fermions (**Fractional** QHE)
    - A Nobel Prize is awarded on this topic this year

- To have more development on **3D QHE**, there are many on going research
    - In **2019**, it was observed on **ZrTe5** **(**Zirconium pentatelluride)
    - What can we exploit from this?
    - We already exploit this much merely on 2D QHE!

Conclusion

- what so special about QHE? Is it important? **YES!!!**
- Quantum Hall effect is powerful, the geometry, material and many other factor won't affect the results
- Relate units with fundamental constants such as ***h* and *e***
- Help us to produce extremely useful materials (Not limited to Stanene)
- Connects various fields of our modern physics and again, another triumph for quantum mechanics

***Yet we emphasize, this is just a very brief introduction for QHE, each of these QHE concepts could take significant amount of time to be fully appreciated**

The End

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