Illustration by Sandbox Studio, Chicago with Corinne Mucha

NEUTRINO OSCILLATION

Measurements of oscillation parameters

θ_{13} and Δm_{23}^2

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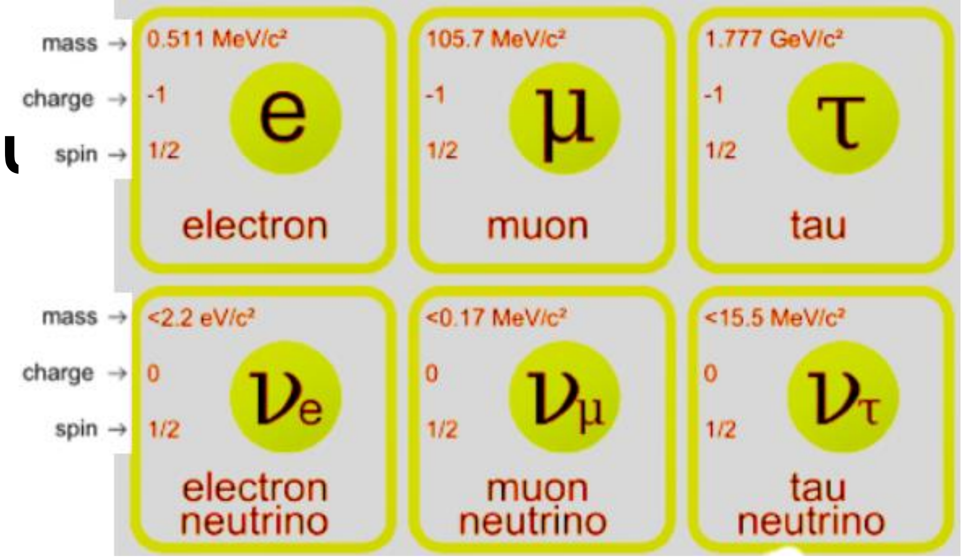
Chan Cassie 1155143734

WHAT ARE NEUTRINOS?

- Denoted by **ν**
- A **lepton**
- Three flavors: **electron**, **muon**, **tau**
- Electrically **neutral**
- Abundant
- Interact very rarely with matter, therefore difficult to detect
- Originally thought to be massless

Lepton family



The Fizzics Organization (2021)

SOLAR-NEUTRINO PROBLEM

Missing neutrinos

- Homestake Experiment in 1960s
- Detect and count neutrinos emitted by solar nuclear fusion
- Only sensitive to **electron neutrinos** $\boldsymbol{\nu_e}$
- Only **~1/3** of prediction was measured

$$4p \rightarrow {}^{4}He + 2e^{+} + 2\nu_e$$

SOLAR-NEUTRINO PROBLEM

Solution

- Super-K and SNO experiments
- Total number of neutrinos is unchanged
- Some neutrinos change flavors,
  i.e. neutrino oscillation

Discovery of neutrino oscillation

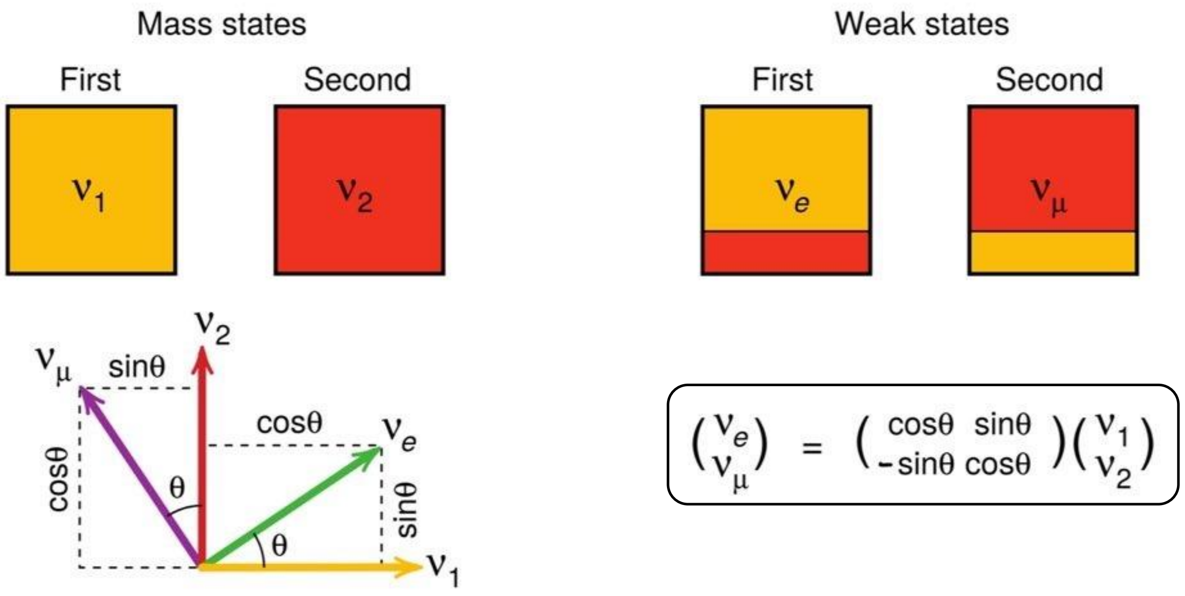
FLAVOR AND MASS EIGENSTATES

2 equivalent ways of describing neutrinos

- Flavor eigenstates
  - $|\nu_e\rangle, |\nu_\mu\rangle, |\nu_\tau\rangle$
  - Correspond to **flavors** detected in **weak interaction**
  - Orthonormal
- Mass eigenstates
  - $|\nu_1\rangle, |\nu_2\rangle, |\nu_3\rangle$
  - Correspond to **masses** useful in describing **propagation**
  - Orthonormal

NEUTRINO MIXING

Simplified case: 2 neutrino mixing



θ : mixing angle

Slansky et al. Los Alamos Sci. 25 (1997)

A flavor (mass) eigenstate is the **superposition** of different mass (flavor) eigenstates.

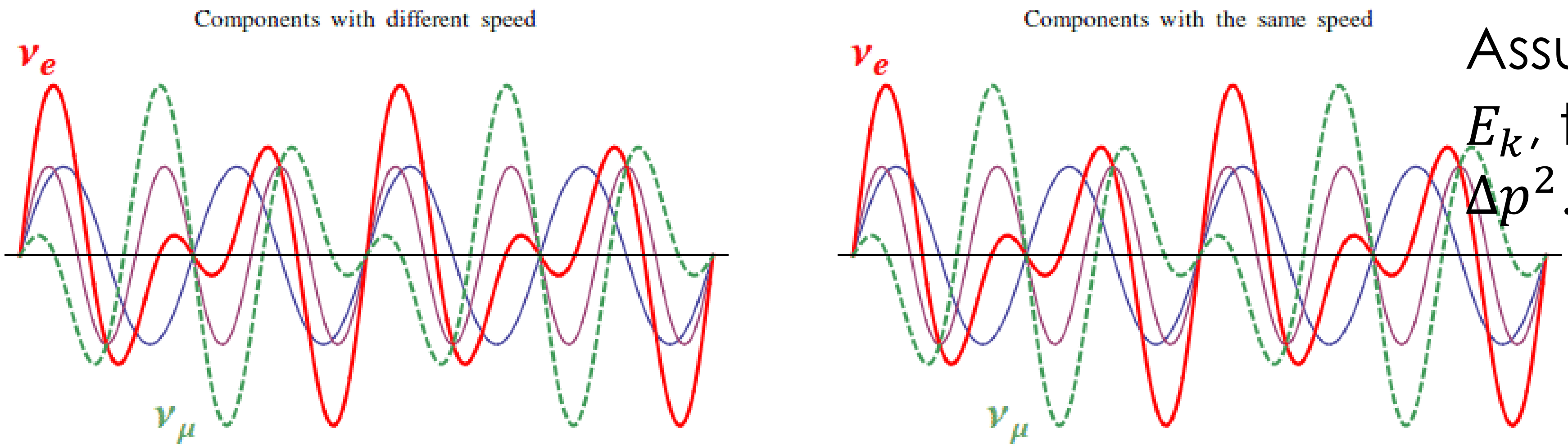
WHY DO NEUTRINOS OSCILLATE?

Propagation of mass eigenstate $|\nu_j\rangle$

$$|\nu_j(x,t)\rangle = e^{i(-E_j t + \vec{p}_j \cdot \vec{x})} |\nu_j(0,0)\rangle$$

$E^2 = m^2 + p^2$, taking $\hbar = c = 1$.

Change in waveform due to phase difference



Assume $E_j = E_k$, then $\Delta m^2 = \Delta p^2$.

Adapted from: http://elusives.eu/outreach/entry/ceaseless-transformation-three-neutrinos.html

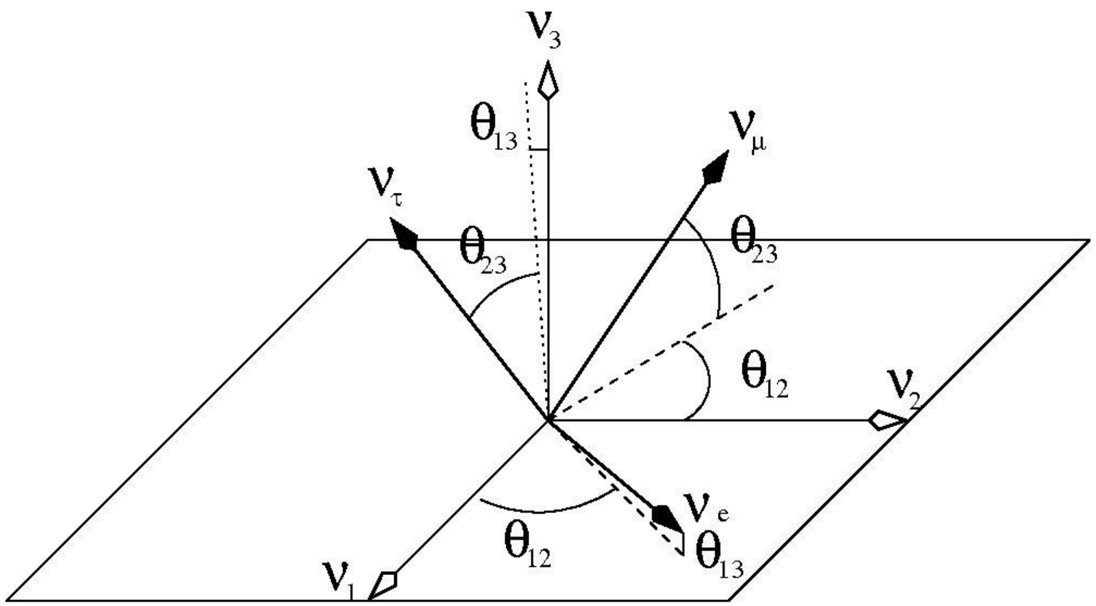
3 NEUTRINO MIXING

PMNS matrix

Flavor eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Mass eigenstates



S. F. King (2007)

PMNS MATRIX

$$U \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\equiv U_{23}U_{13}U_{12}$$

where $s_{ij} = sin\theta_{ij}$, $c_{ij} = cos\theta_{ij}$.

θ_{ij}: mixing angle
δ_{CP}: CP phase

Properties

- Complex
- Unitary
- Can be decomposed to 3 2D rotations

IMPLICATION AND IMPORTANCE

- Neutrino oscillation arises from phase difference between mass states, leading to change in state composition during propagation.
- $\Delta m^2 \neq 0 \Rightarrow \boldsymbol{Neutrinos\ have\ mass}$!!
- 2015 Nobel Prize for the discovery of neutrino oscillation
- Possible existence of right handed neutrinos

WHAT TO LOOK FOR?

- Flavor states dependence on each mass state, governed by mixing angles θ
- Mass differences between the mass eigenstates
- CP violation for non-zero δ_{CP}

MEASUREMENTS OF OSCILLATION PARAMETERS θ_{13} AND Δm^2_{23}

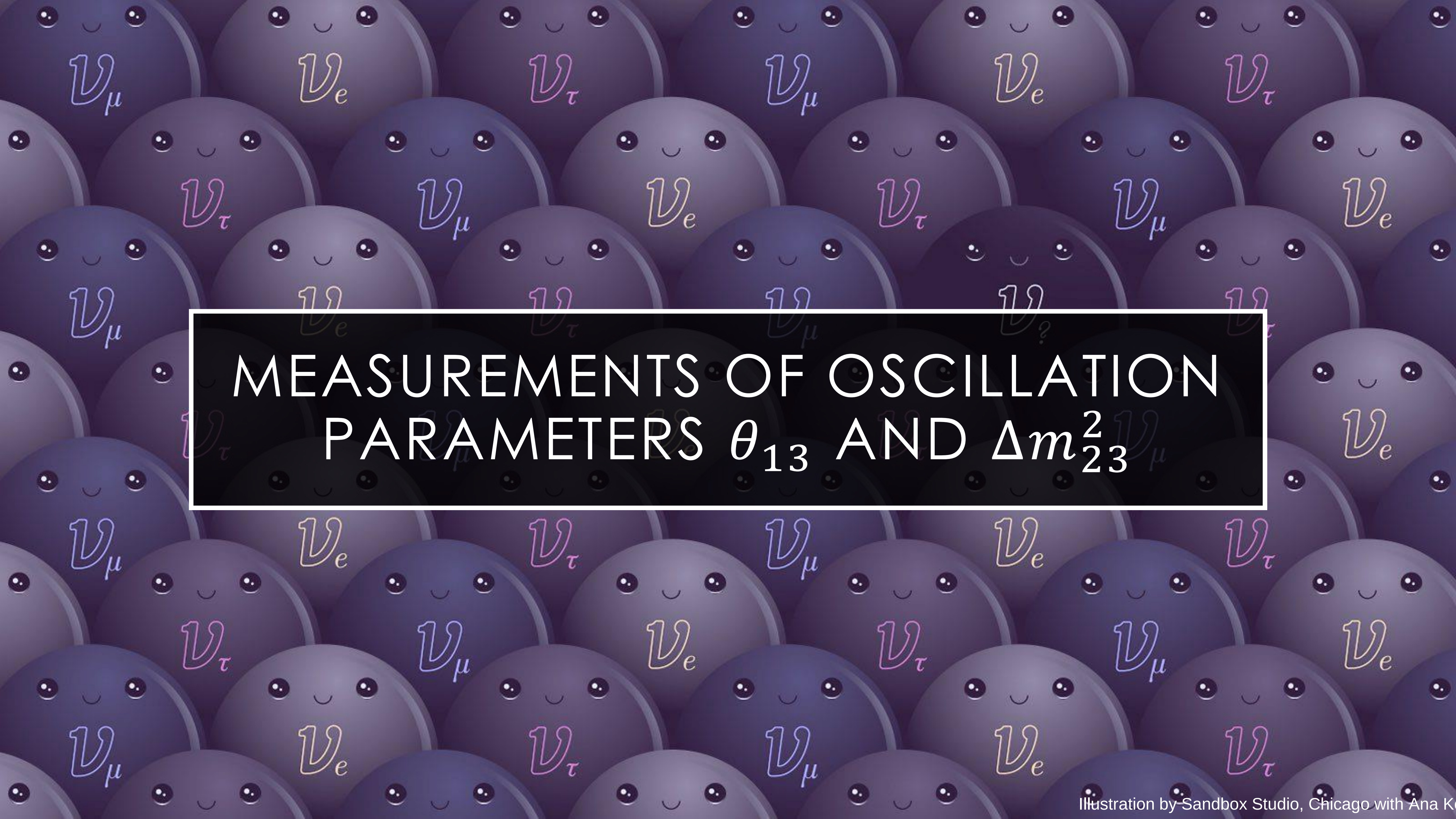


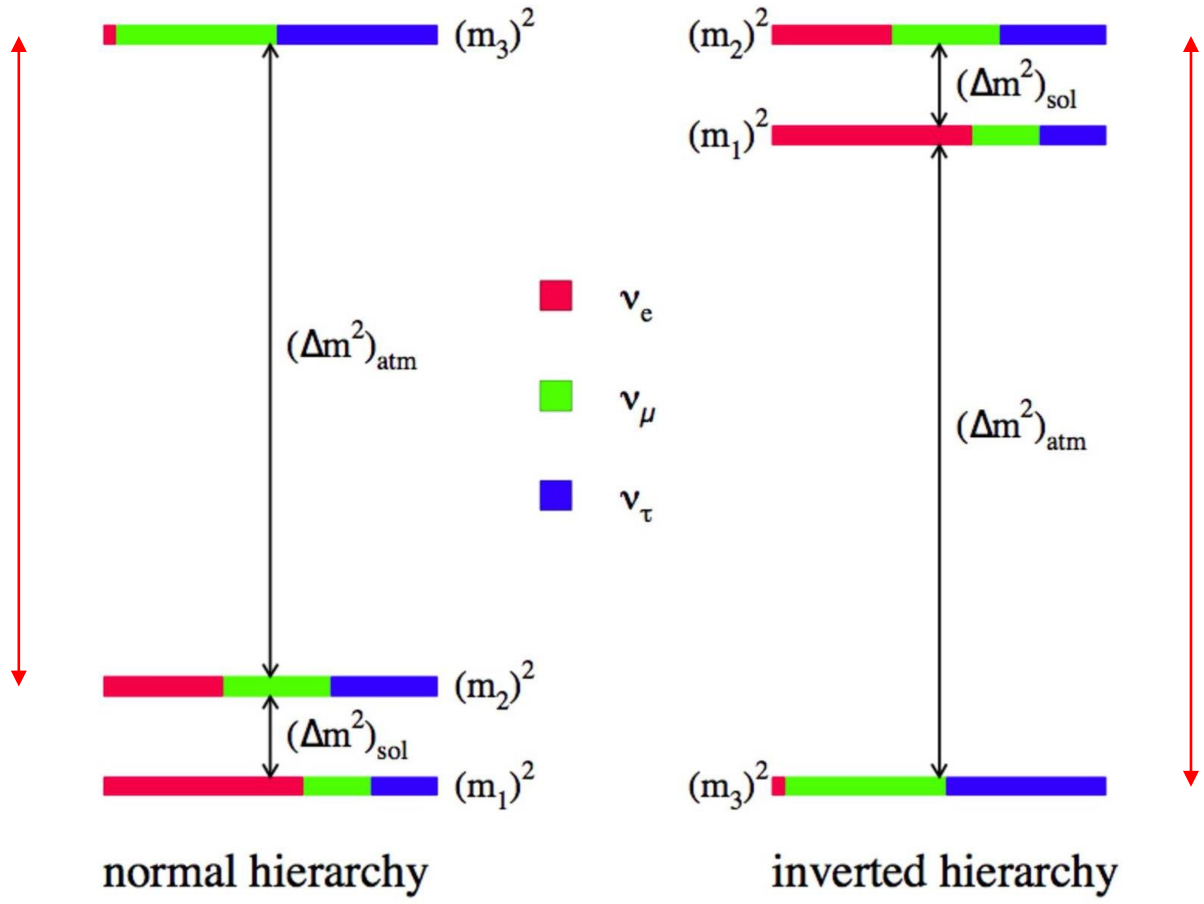
Illustration by Sandbox Studio, Chicago with Ana K

NEUTRINO MASS HIERARCHY

Difference in Δm_{23}^2 due to mass ordering

$$\Delta m_{ij}^2 = m_i^2 - m_j^2$$

Δm_{32}^2



Δm_{23}^2

R. N. Cahn et al. (LBL, Berkeley) (2013)

OSCILLATION PROBABILITY

$$P\left(\nu_\alpha(0,0) \to \nu_\beta(x,t)\right) = \left|\langle \nu_\beta(x,t) | \nu_\alpha(0,0) \rangle\right|^2$$

Simplified case: 2 neutrino mixing

$$P_{\alpha\beta}(L) = sin^2(2\theta) sin^2(\frac{\Delta m^2 L}{4E_\nu})$$

L: baseline

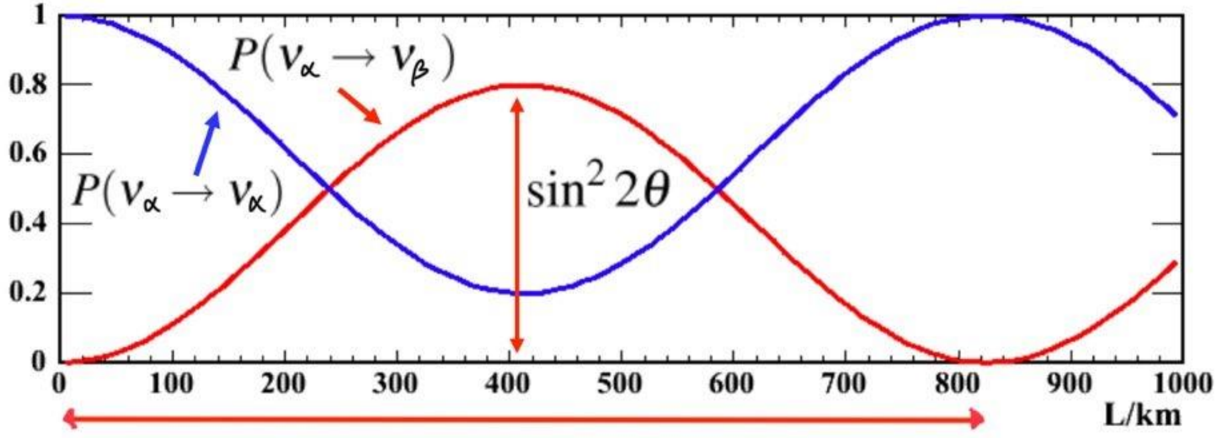
taking $\hbar = c = 1$.

If $\Delta m^2 = 0$,
then $P\left(\nu_\alpha \to \nu_\beta\right) = 0$.

OSCILLATION PROBABILITY

$$P_{\alpha\beta}(L) = sin^2(2\theta)sin^2(\frac{\Delta m^2 L}{4E_\nu}) \qquad P_{\alpha\alpha}(L) = 1 - P_{\alpha\beta}(L)$$

$$\Delta m^2 = 0.003\,\mathrm{eV}^2, \quad \sin^2 2\theta = 0.8, \quad E_\nu = 1\,\mathrm{GeV}$$



$$\lambda_{osc} = \frac{4\pi E}{\Delta m^2}$$

First oscillation extremum

$$\frac{L}{E} = \frac{\lambda_{osc}}{2}$$

at which the detector should be set

Reproduced from Mark Thomson (2011)

$\bar{\nu}_e$ DISAPPEARANCE

$\bar{\boldsymbol{\nu}}_{\boldsymbol{e}}$ **survival probability** for small L/E and $\delta_{CP} = 0$

$$P_{ee} \approx 1 - sin^2 2\theta_{13} sin^2 \Delta_{ee}$$

where $\Delta_{ee} = cos^2\theta_{12} sin^2\Delta_{31} + sin^2\theta_{12} sin^2\Delta_{32}$

and $\Delta_{ij} = 1.267\Delta m^2_{ij}\frac{L}{E}$ for $i, j = \{1, 2, 3\}$.

Δ_{ee}: effective mass square differen

Short-baseline reactor experiment

- Daya Bay, RENO and Double Chooz

REACTOR EXPERIMENT

- Nuclear reactors produce intense flux of $\bar{\nu}_e$
- Liquid scintillator loaded with Gadolinium Gd
  - Low energy threshold
- Inverse beta decay

$$\bar{\nu}_e + p^+ \rightarrow e^+ + n$$

- Positron annihilation

$$e^+ + e^- \rightarrow 2\gamma$$

E_{prompt} : **prompt** e^+ energy deposition

- **Delayed** neutron capture

$$n + Gd \rightarrow Gd^* \rightarrow Gd + \gamma's\ (8\ MeV)$$

REACTOR EXPERIMENT

Neutrino energy measurement

$$E_{\overline{\nu}_e} = E_{prompt} + 0.78\ MeV$$

Near-far cancellation

$$\frac{N_f}{N_n} = \left(\frac{N_{p,f}}{N_{p,n}}\right)\left(\frac{L_f}{L_n}\right)^{-2}\left(\frac{\varepsilon_f}{\varepsilon_n}\right)\left[\frac{P_{ee}(E_\nu, L_f)}{P_{ee}(E_\nu, L_n)}\right]$$

f: far
n: near
p: proton

- To overcome uncertainty in the $\bar{\nu}_e$ flux emitted by reactors

Roy Kaltschmidt (LBL, Berkele

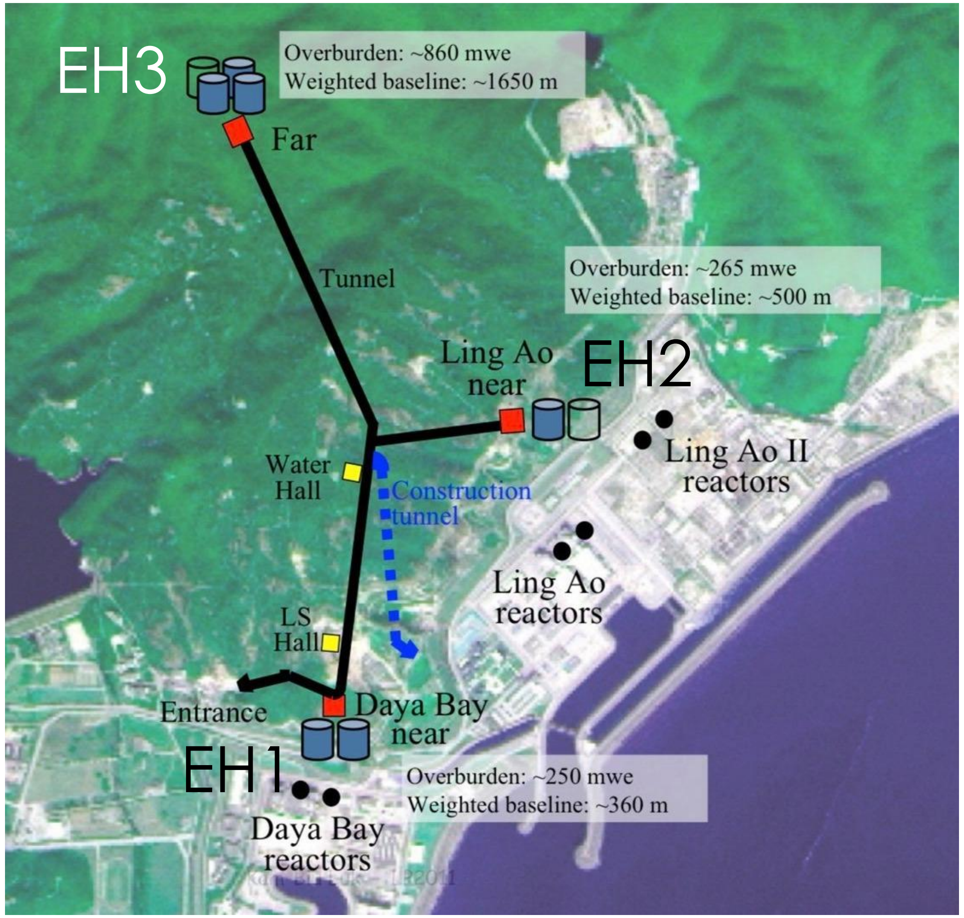
DAYA BAY EXPERIMENT

$E_{\bar{\nu}_e} \sim 4\ MeV$
$L_n \sim 500\ m$
$L_f \sim 1.6\ km$

Near detectors: EH1, EH2

Far detectors: EH3

Schematic of Daya Bay Experiment



F. P. An *et al.* (Daya Bay Collaboration) (2017)

CONTROL OF RELATIVE UNCERTAINTIES

$\frac{N_f}{N_n}$: relative uncertainty minimized by signal selection

$\frac{N_{p,f}}{N_{p,n}}$: identical detectors, <0.5% difference in N_p

$\frac{\varepsilon_f}{\varepsilon_n}$: potential difference reduced by energy calibration

ε: estimated in simulations of signal selection

$$\frac{N_f}{N_n} = \left(\frac{N_{p,f}}{N_{p,n}}\right)\left(\frac{L_f}{L_n}\right)^{-2}\left(\frac{\varepsilon_f}{\varepsilon_n}\right)\left[\frac{P_{ee}(E_\nu, L_f)}{P_{ee}(E_\nu, L_n)}\right]$$

θ_{13} FROM RATE DEFICIT

$$\widetilde{N}_{exp}^{EH3} = w^{EH1}\widetilde{N}_{obs}^{EH1} + w^{EH2}\widetilde{N}_{obs}^{EH2}$$

where $w^{EH1} = 0.05545$ and $w^{EH2} = 0.2057$.

exp: expected assuming **no** oscillation
obs: observed

$$R = \frac{\widetilde{N}_{obs}^{EH3}}{\widetilde{N}_{exp}^{EH3}}$$
$$= 0.949 \pm 0.002(stat.) \pm 0.002(syst.)$$
$$< 1$$

Evidence of $\bar{\nu}_e$ disappearance

θ_{13} FROM RATE DEFICIT

For experimental hall k,

$N^k_{pred} = N^k_{no\ osc} P^k_{ee}, \qquad P^k_{ee} = 1 - \eta^k sin^2 2\theta_{13}$

$$R = \frac{N^{EH3}_{pred}}{w^{EH1} N^{EH1}_{pred} + w^{EH2} N^{EH2}_{pred}}$$

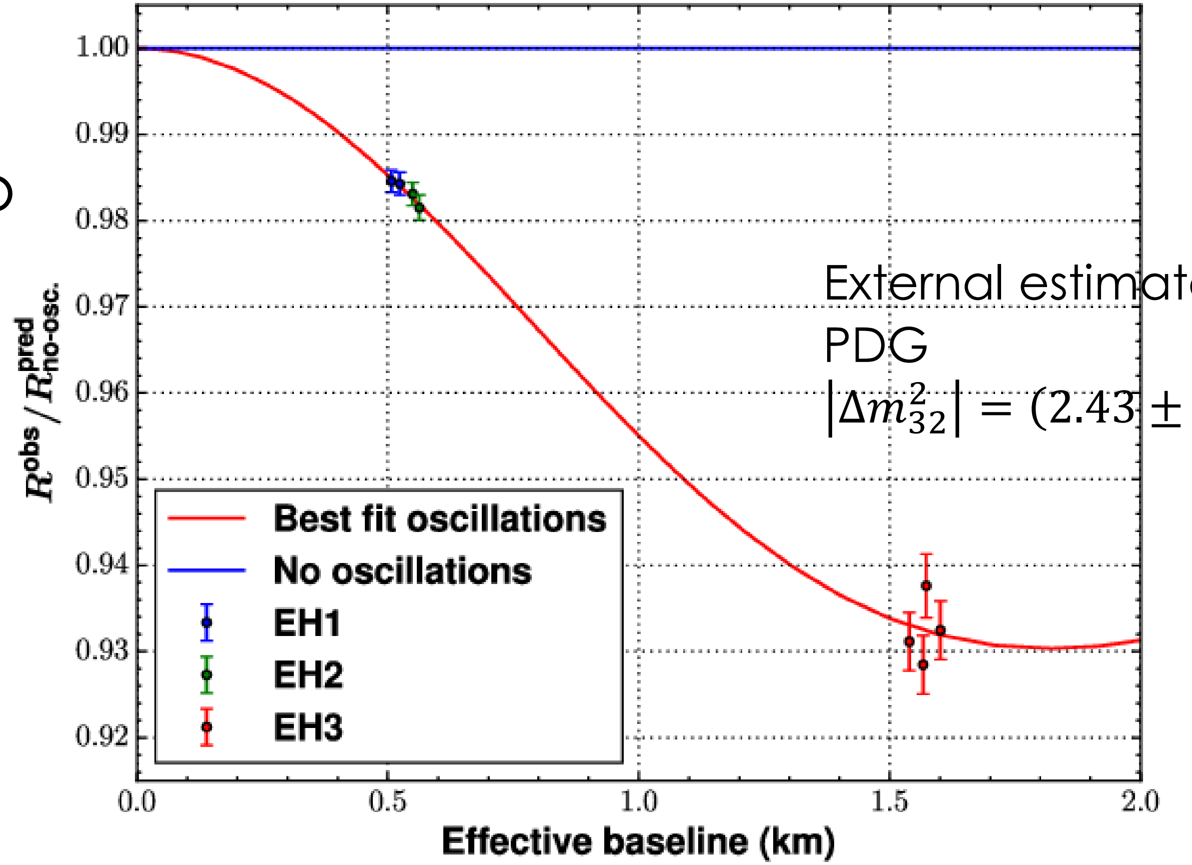
$$= \frac{1 - \eta^{EH3} sin^2 2\theta_{13}}{1 - \eta^n sin^2 2\theta_{13}}$$

where $\eta^n = \frac{1}{N^{EH3}_{no\ osc}} (w^{EH1} N^{EH1}_{no\ osc} \eta^{EH1} + w^{EH2} N^{EH2}_{no\ osc} \eta^{EH2})$.

$$sin^2 2\theta_{13} = 0.085 \pm 0.003(stat.) \pm 0.003(syst.)$$

θ_{13} FROM STATISTICAL METHOD

y: Ratio of the detected signal to that expected assuming no oscillation



External estimate of $|\Delta m^2_{32}|$ from PDG

$|\Delta m^2_{32}| = (2.43 \pm 0.07) \times 10^{-3} eV^2$

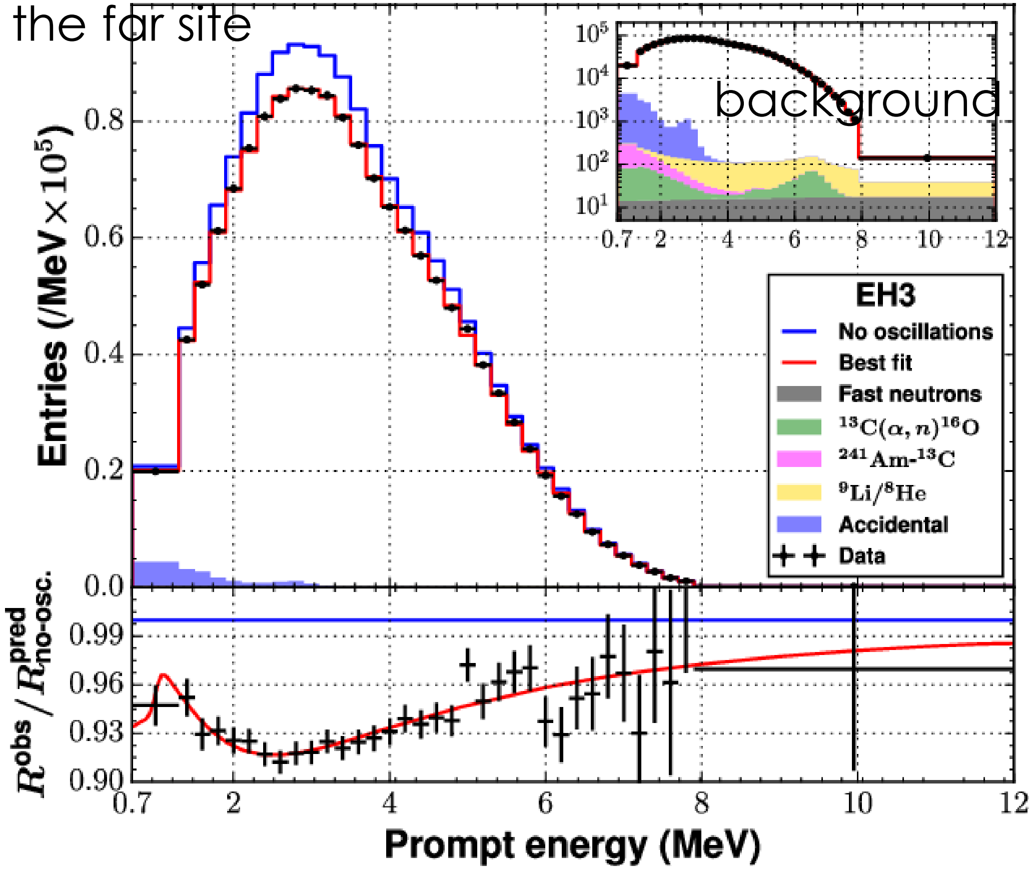
F. P. An *et al.* (Daya Bay Collaboration) (2017)

$$sin^2 2\theta_{13} = 0.0850 \pm 0.0030(stat.) \pm 0.0028(syst.)$$

$$\frac{\chi^2}{ndf} = 0.85$$

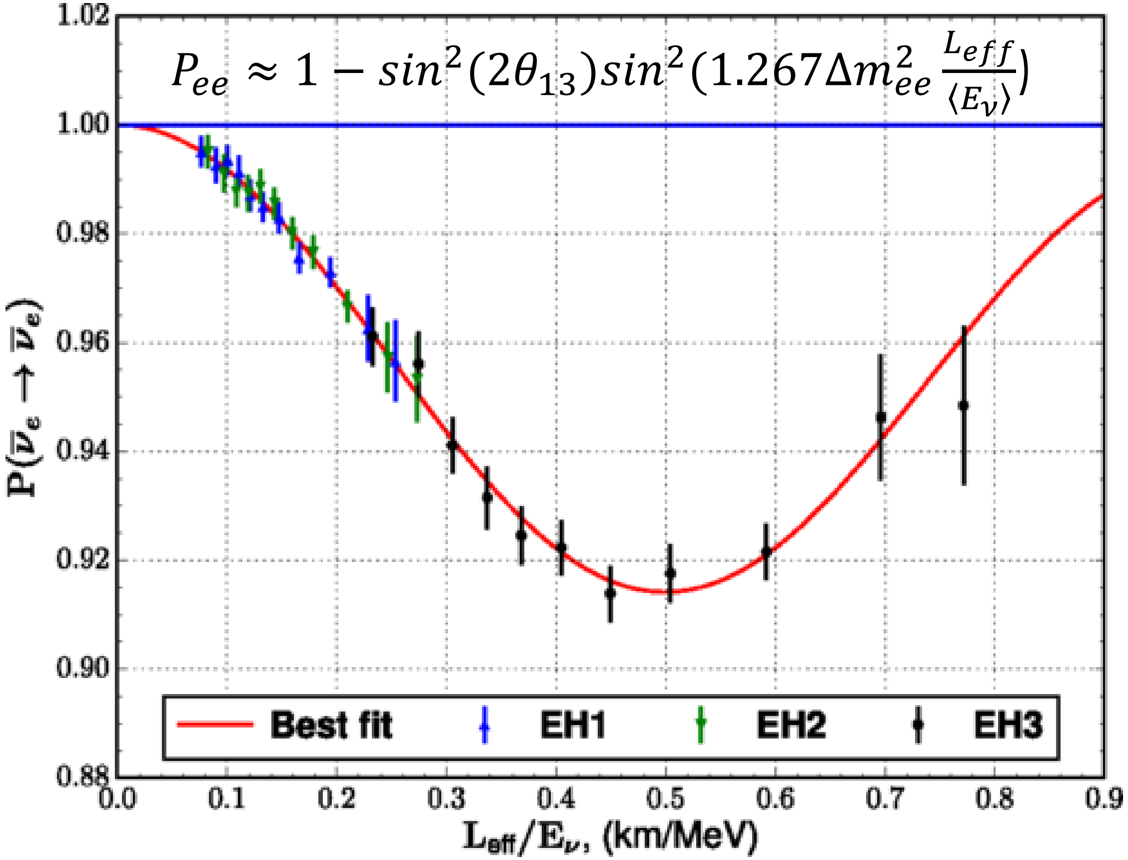
Δm^2_{23} FROM ENERGY SPECTRA

Background subtracted E_{e^+} spectrum at the far site



Ratio of signal to prediction assuming no oscillation

$\bar{\nu}_e$ survival probability from near and far detectors da

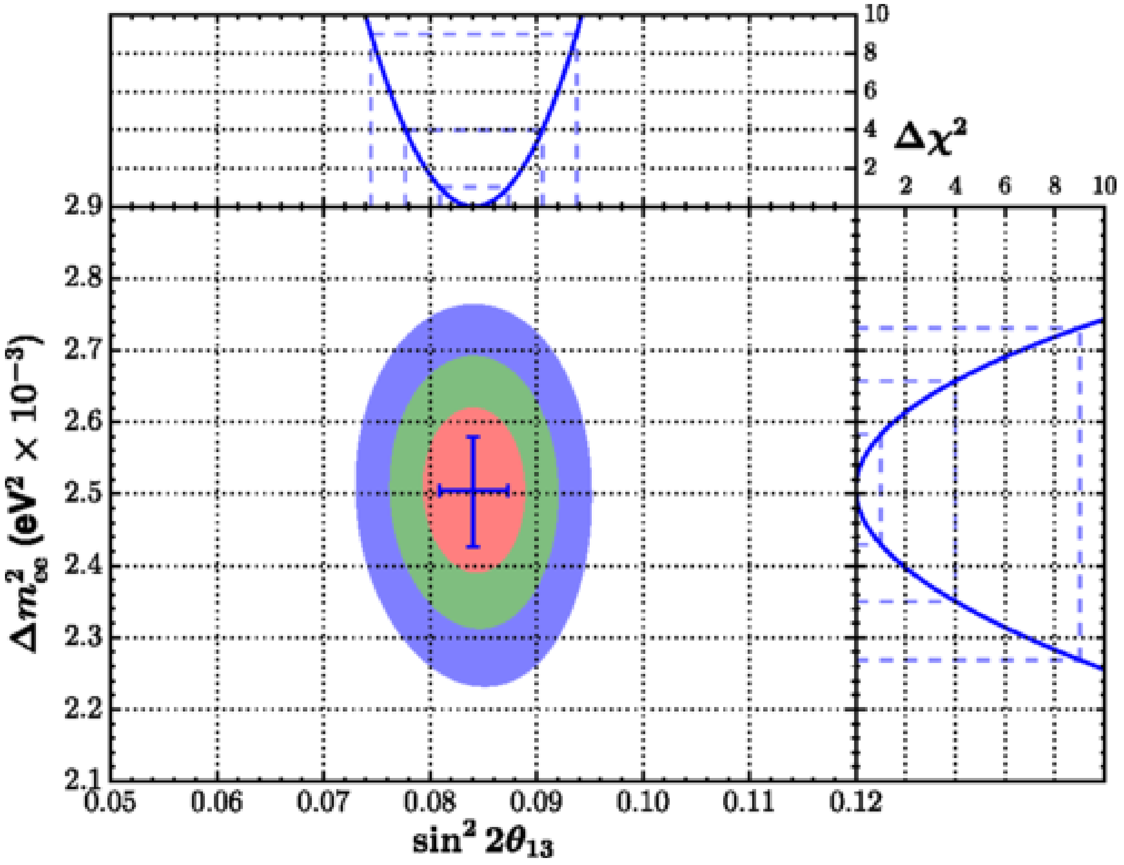


$\langle \boldsymbol{E_\nu} \rangle$: average $\boldsymbol{E_\nu}$ estimated for each bin in prompt positron spectra

F. P. An *et al.* (Daya Bay Collaboration) (2015)

RESULTS

2D confidence regions of $sin^2 2\theta_{13}$ and $|\Delta m^2_{ee}|$



Red: 1σ Green: 2σ Blue: 3σ
Error bars:: 1D 1σ confidence interval

$$\sin^2 2\theta_{13} = 0.0841 \pm 0.0027(\text{stat.}) \pm 0.0019(\text{syst.}),$$

$$|\Delta m^2_{ee}| = [2.50 \pm 0.06(\text{stat.}) \pm 0.06(\text{syst.})] \times 10^{-3}\,\text{eV}^2,$$

$$\Delta m^2_{32}(\text{NH}) = [2.45 \pm 0.06(\text{stat.}) \pm 0.06(\text{syst.})] \times 10^{-3}\,\text{eV}^2,$$

$$\Delta m^2_{32}(\text{IH}) = [-2.56 \pm 0.06(\text{stat.}) \pm 0.06(\text{syst.})] \times 10^{-3}\,\text{eV}^2,$$

$$\frac{\chi^2}{\text{NDF}} = \frac{234.7}{280 - 17} = 0.89.$$

F. P. An *et al.* (Daya Bay Collaboration) (2017)

ν_e APPEARANCE

For small L/E,

$$P(\nu_\mu \to \nu_e) \approx \sin^2(2\theta_{13})\sin^2(\theta_{23})\sin^2\Delta_{32} \mp \Delta_{21} 8 J_{CP} \sin^2\Delta_{32}$$

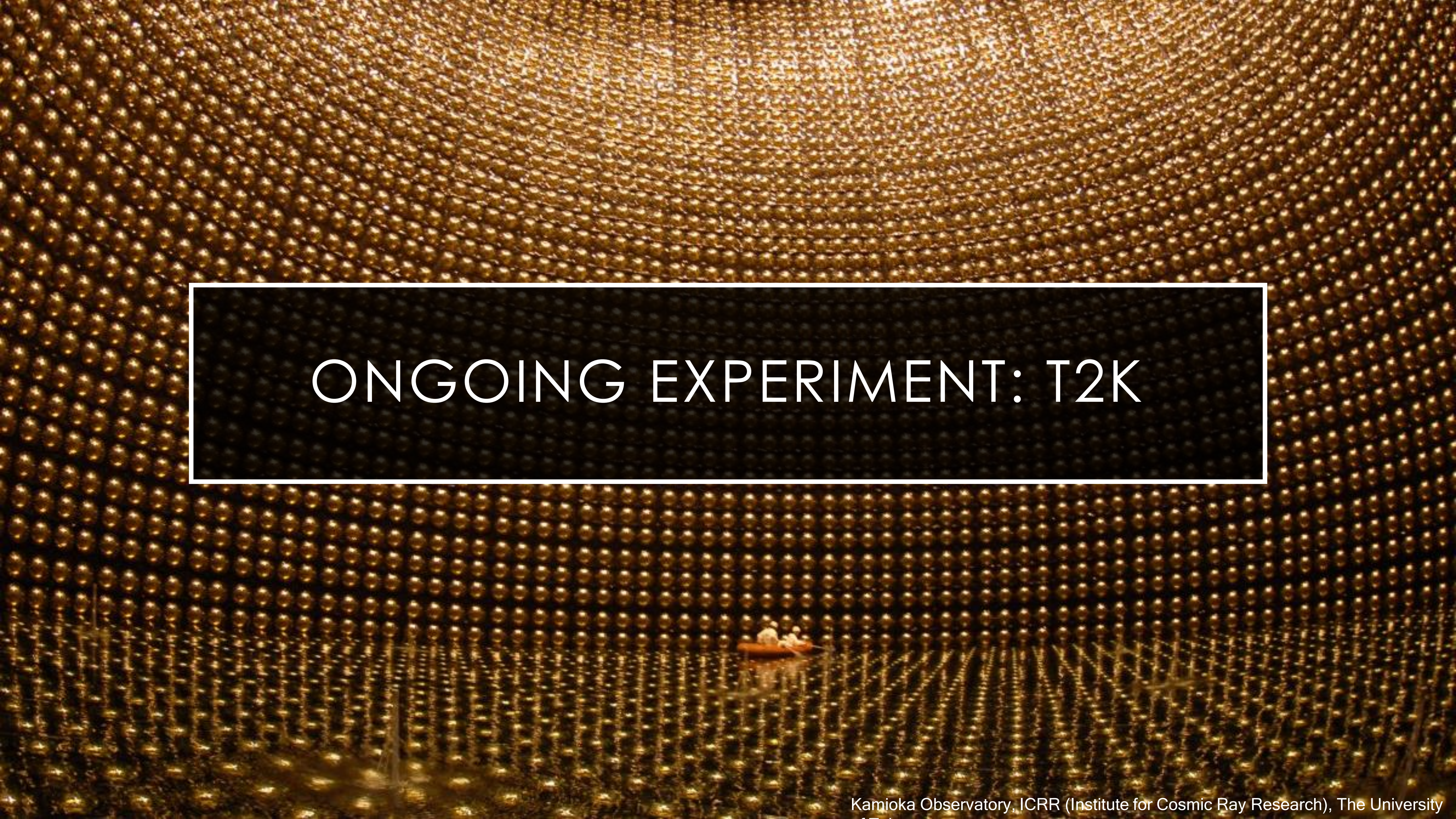
where $J_{CP,l} = \frac{1}{8}\cos\theta_{13}\sin(2\theta_{12})\sin(2\theta_{23})\sin(2\theta_{13})\sin(\delta_{CP})$

(neglecting effects from propagation through matter)

Long-baseline accelerator experiment

- T2K, NOvA, MINOS

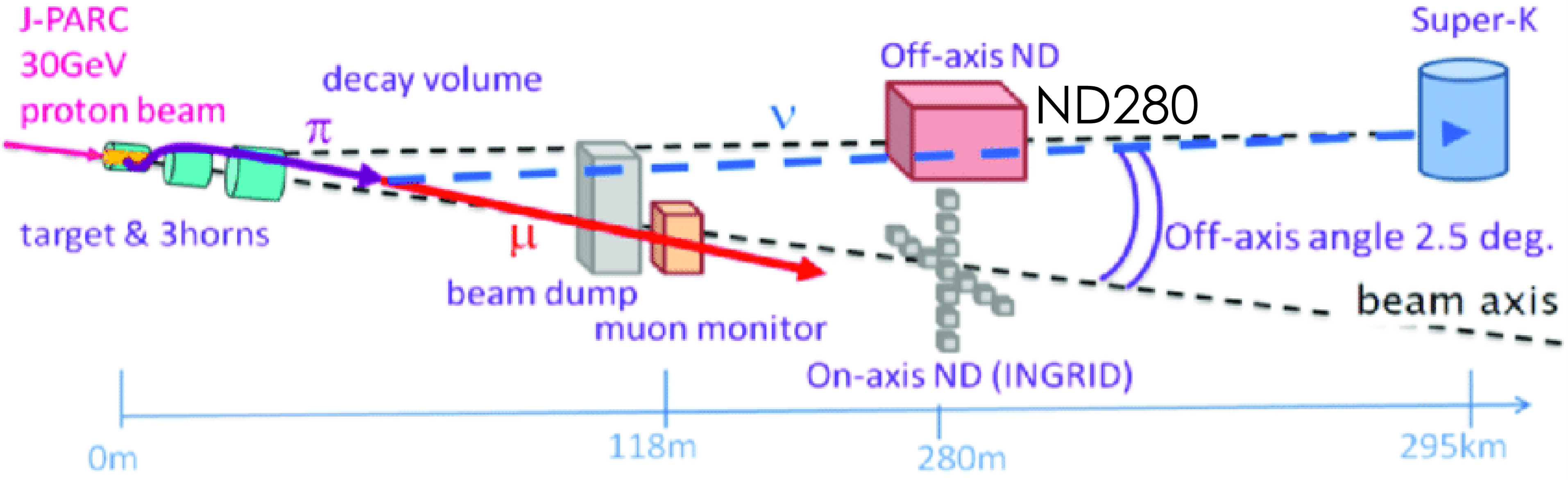
ONGOING EXPERIMENT: T2K



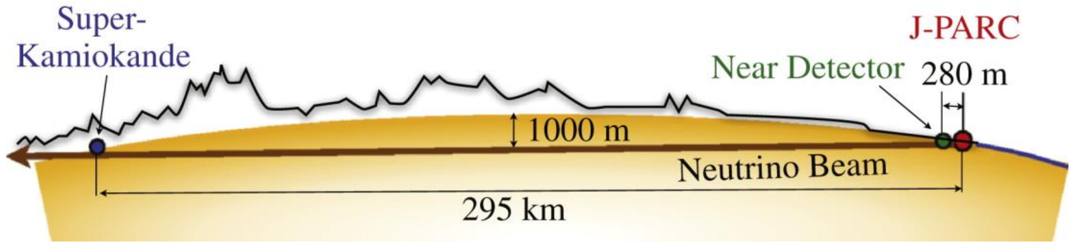
Kamioka Observatory, ICRR (Institute for Cosmic Ray Research), The University

T2K EXPERIMENT

Schematic of T2K Experiment



M. Khabibullin (2018)



S. Assylbekov *et al.* (2012)

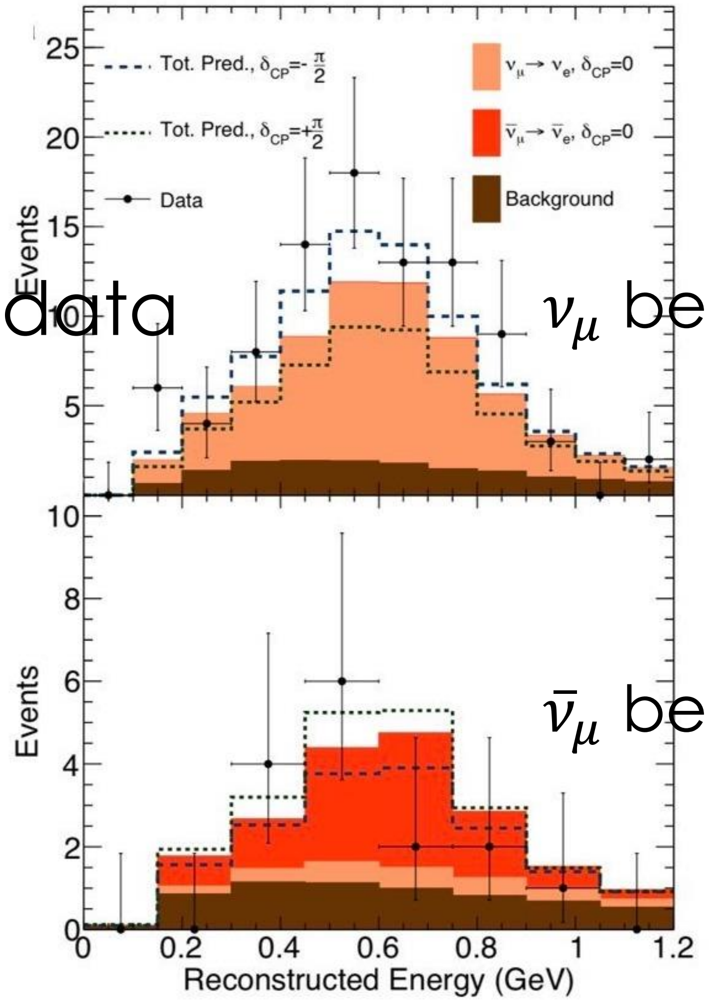
T2K: Tokai to Kamioka

RESULTS

Marginal likelihood fit to far detector data

$\Delta m_{32}^2 = (2.45 \pm 0.07) \times 10^{-3} eV^2/c^4$ (NH)

$\Delta m_{13}^2 = (2.43 \pm 0.07) \times 10^{-3} eV^2/c^4$ (IH)



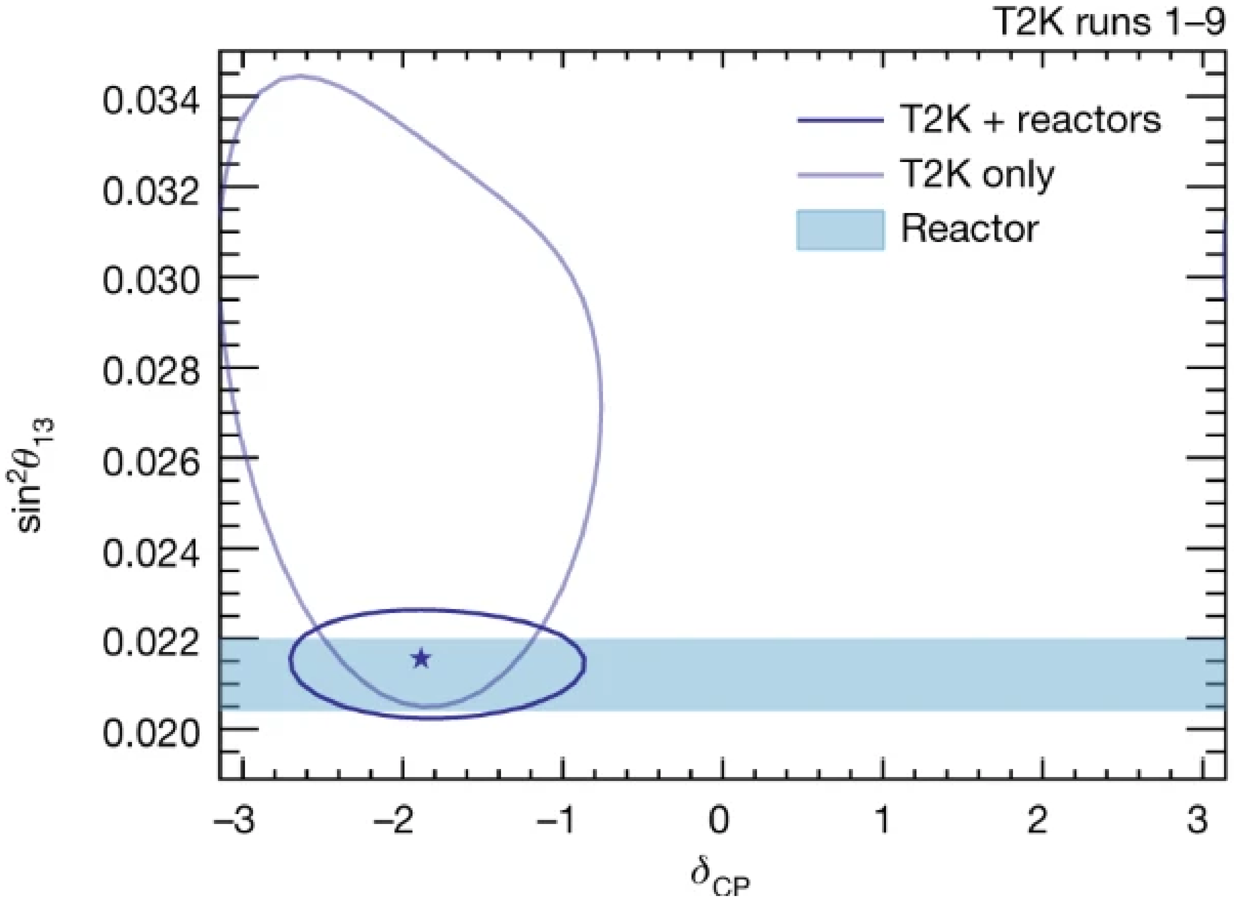
ν_μ beam

$\bar{\nu}_\mu$ beam

The T2K Collaboration (2020)

RESULTS

2D confidence region at 68.27% CL for $sin^2\theta_{13}$ vs δ_{cp}



The T2K Collaboration (2020)

OSCILLATION PARAMETERS (Global Fit)

with SK atmospheric data	Normal Ordering (best fit) bfp $\pm 1\sigma$	Normal Ordering (best fit) 3σ range	Inverted Ordering ($\Delta\chi^2 = 7.1$) bfp $\pm 1\sigma$	Inverted Ordering ($\Delta\chi^2 = 7.1$) 3σ range
$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.573^{+0.016}_{-0.020}$	$0.415 \to 0.616$	$0.575^{+0.016}_{-0.019}$	$0.419 \to 0.617$
$\theta_{23}/^\circ$	$49.2^{+0.9}_{-1.2}$	$40.1 \to 51.7$	$49.3^{+0.9}_{-1.1}$	$40.3 \to 51.8$
$\sin^2\theta_{13}$	$0.02219^{+0.00062}_{-0.00063}$	$0.02032 \to 0.02410$	$0.02238^{+0.00063}_{-0.00062}$	$0.02052 \to 0.02428$
$\theta_{13}/^\circ$	$8.57^{+0.12}_{-0.12}$	$8.20 \to 8.93$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.96$
$\delta_{CP}/^\circ$	197^{+27}_{-24}	$120 \to 369$	282^{+26}_{-30}	$193 \to 352$
$\frac{\Delta m^2_{21}}{10^{-5}\ eV^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ eV^2}$	$+2.517^{+0.026}_{-0.028}$	$+2.435 \to +2.598$	$-2.498^{+0.028}_{-0.028}$	$-2.581 \to -2.414$

I. Esteban *et al.* (2020)

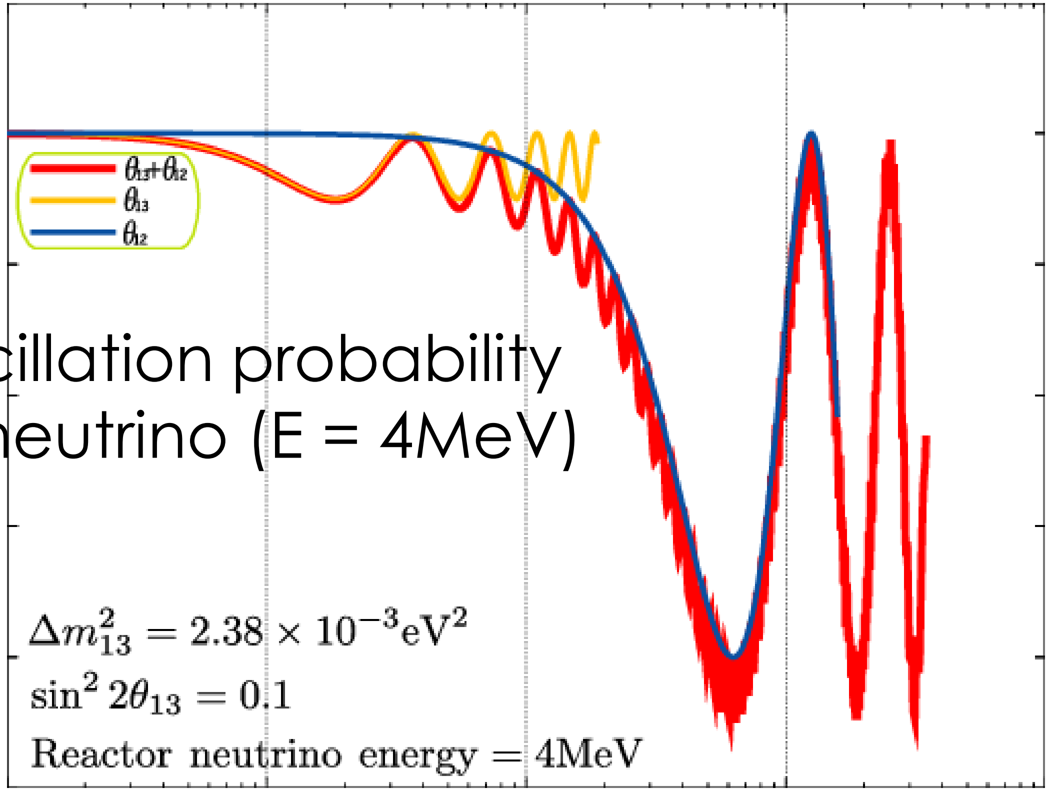
SUMMARY

- Neutrino oscillation shows that neutrinos have mass
- Oscillation probability tells the value of oscillation parameters
  - θ_{13} and Δm_{23}^2 from $\bar{\nu}_e$ disappearance
- Near-far cancellation method in reactor experiments to reduce uncertainty in neutrino flux
- Recent experiments use pure neutrino beam to detect neutrino appearance / disappearance

APPENDIX

$\bar{\nu}_e$ survival probability for small L/E

$$P_{ee} = 1 - sin^2 2\theta_{13} sin^2\Delta_{ee} - cos^4\theta_{13} sin^2 2\theta_{12} sin^2\Delta_{21}$$



Neutrino oscillation probability for reactor neutrino (E = 4MeV)

T. Konno et al. (2009)

APPENDIX

Daya Bay Experiment

- Effective baseline L_{eff}

$$\int sin^2\left(\Delta m_{ee}^2 \frac{L_{eff}}{E_\nu}\right)\frac{dN_o}{dE_\nu}dE_\nu = \sum_j^{reactors} sin^2\left(\Delta m_{ee}^2 \frac{L_j}{E_\nu}\right)\frac{dN_o^j}{dE_\nu}dE_\nu$$

where $\frac{dN_o^j}{dE_\nu}$ is the expected signal in a given reactor due to reactor to j assuming no neutrino oscillation.

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