Test of Attractive and Repulsive Casimir Effect

Group 15:

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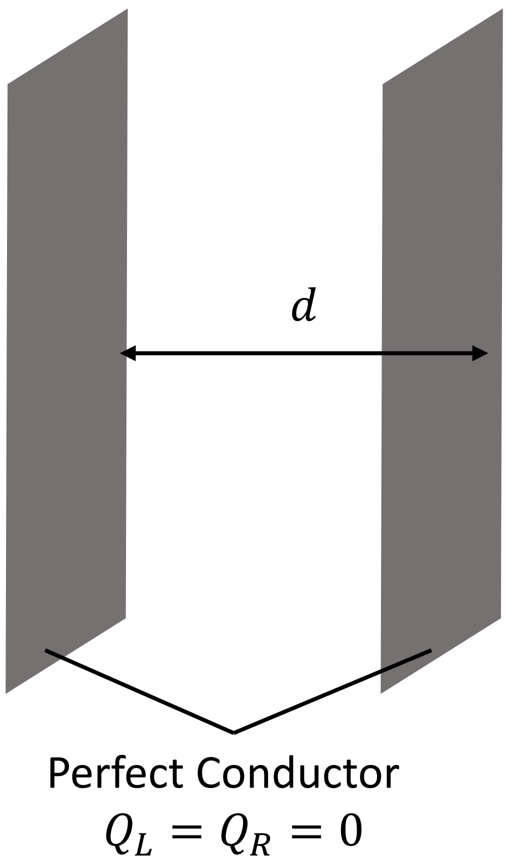
Introduction: Thought Experiment

- 2 infinitely large parallel plates of perfect conductor
- Separation: d
- No electric charge
- Ignore gravitational effect

- Zero force?
- Wrong!
- Casimir force



Introduction: Vacuum Fluctuation & Casimir Effect

- Quantum Field Theory:
  - No true vacuum
  - Always Quantum fields fluctuating

- Conductor
- ⇒ E-field at plates equals to 0
- Restricted oscillation modes between plates
- ⇒ Energy difference
- ⇒ Force (Casimir effect)

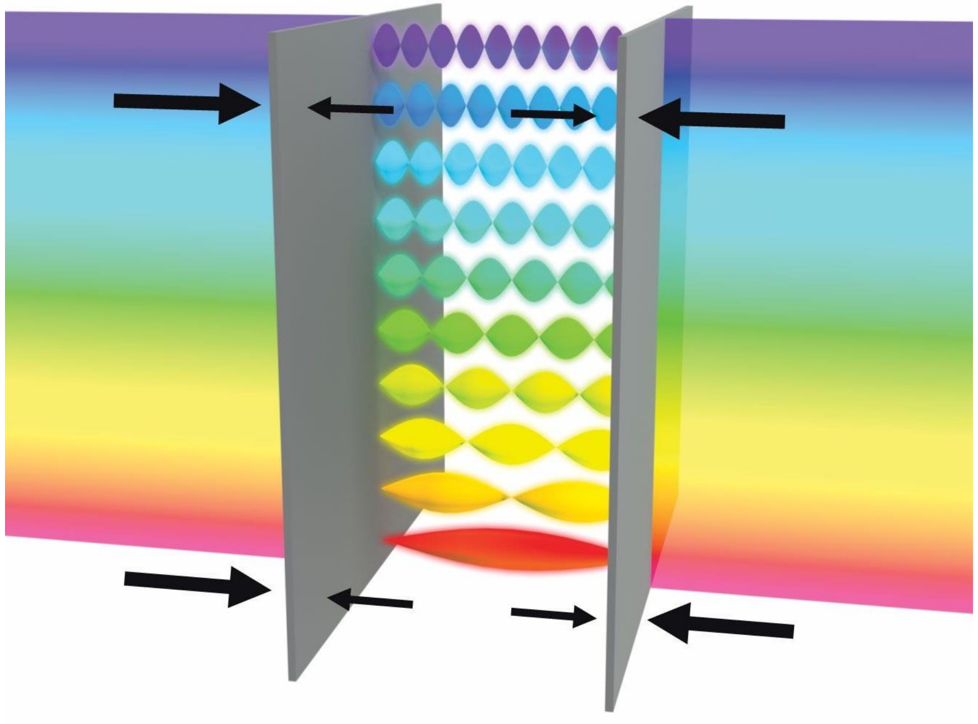


Illustration of the fluctuating E-field with the plates
Adapted from: Stange, Campbell, Bishop (2021)

Theory

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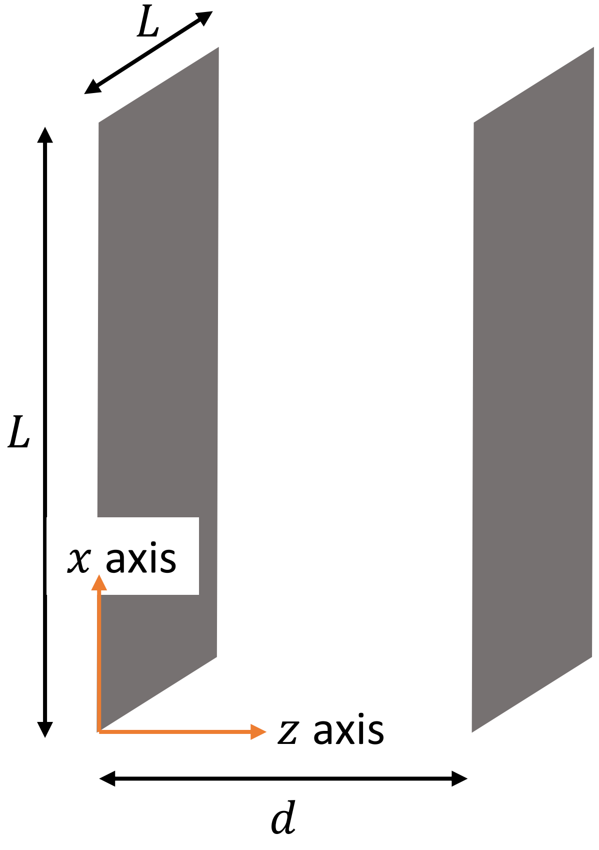
Introduction: Brief Derivation of Casimir Effect

- Allowed oscillation modes between plates:
- x, y direction: Periodic BC (PBC) with cell size of $L \times L$:
- PBC: $\mathbb{E} \propto e^{ik_x x}$ and $\mathbb{E}(x = 0) = \mathbb{E}(x = D)$
- $\mathbb{E}$: Electric field
- k_i: i component of the wavevector
- $\Rightarrow$ $k_x = \frac{2n\pi}{L}, n = 0, \pm 1, \pm 2, \ldots$ (1)
- Similarly, $k_y = \frac{2n\pi}{L}, n = 0, \pm 1, \pm 2, \ldots$ (2)
- z direction: Conductor BC:$\mathbb{E}(z = 0) = \mathbb{E}(z = L) = 0$
- $E \propto \sin(k_z z) \Rightarrow$ $k_z = \frac{m\pi}{d}, m = 1, 2, 3, \ldots$ (3)



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Introduction: Brief Derivation of Casimir Effect

- Energy of 1 mode: $E_{mode} = \hbar\omega = \hbar c|\vec{k}| = \hbar c\left(k_x^2 + k_y^2 + k_z^2\right)^{1/2}$
- Energy between plates: $E = \hbar c \sum_{k_x} \sum_{k_y} \sum_{k_z} \left(k_x^2 + k_y^2 + k_z^2\right)^{1/2}$
- Changing $\sum_{k_x/k_y}$ into $\int_{-\infty}^{\infty} dk_x/dk_y$ using (1) and (2)
- Substitute (3) and doing some simplification:

$$E = \frac{L^2 \hbar c \pi^2}{4L^3} \sum_{m=1}^{\infty} \int_0^{\infty} (u + m^2)^{1/2} du \qquad (4)$$

- u is some variable to simplified the expression

Introduction: Brief Derivation of Casimir Effect

- Energy density of vacuum: PBC in x, y **and z direction**
- Similarly, $E = \sum E_{mode}$ & change all $\sum$ to $\int dk$
- $E_{free} = L^2 d \frac{\hbar c}{(2\pi)^3} \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \int_{-\infty}^{\infty} dk_z \sqrt{k_x^2 + k_y^2 + k_z^2}$
- Simplifying,

$$E_{free} = \frac{L^2 \hbar c \pi^2}{4d^3} \int_0^{\infty} \int_0^{\infty} (u + x^2)^{1/2} du\, dx \qquad (5)$$

c.f.

$$E = \frac{L^2 \hbar c \pi^2}{4d^3} \sum_{m=1}^{\infty} \int_0^{\infty} (u + m^2)^{1/2} du \qquad (4)$$

Introduction: Brief Derivation of Casimir Effect

- Energy difference: $\Delta E = E_{free} - E$

$$\Delta E = \frac{L^2 \hbar c \pi^2}{720 d^3} \qquad (6)$$

- Casimir Force: $F = \frac{\partial(\Delta E)}{\partial d}$
- $F = -\frac{L^2 \hbar c \pi^2}{240 d^4}$

$$\frac{F}{Area} = -\frac{\hbar c \pi^2}{240 d^4} \qquad (7)$$

- Attractive, inverse quartic

Difference between the Van der Waals force and the Casimir Force

- Treat the parallel plates as the “molecules”

- Parallelly obtain the Casimir force from the study of Van der Waals force

- The derivation later for the repulsive Casimir force comes from the book called “The general theory of Van der Waals forces”

Difference between the Van der Waals force and the Casimir Force

- When the distance between the two molecules or particles is getting larger (typically to more than a few nanometres)
  - ➢ Electrostatic interaction **no longer instantaneous** (FINITE speed of light)
  - ➢ **Retardation effect**

- Mathematically, the retardation effect **become important when $\boldsymbol{\lambda \gtrsim R}$**, where $\lambda = hc/E_t$, E_t is the energy of the corresponding transitions between the ground state and the excited states of the atoms, R is the separation between two atoms (Adv. Phys. 10, 165–209 (1961))

Difference between the Van der Waals force and the Casimir Force

Consider the perturbation energy term,

- For $R \ll \lambda$, $\Delta_4 E$ can be ignored (only up to $\Delta_2 E$ is considered for Van der Waals force)
- BUT **for $\boldsymbol{\lambda \gtrsim R}$,** $\Delta_4 E$ **CANNOT be neglected**.

$\Delta_4 E$: the fourth order perturbation energy due to the interaction between the two atoms

$\Delta_2 E$: the second order perturbation term

(From Casimir and Polder (Phys. Rev. 73, 360–372 (1948)))

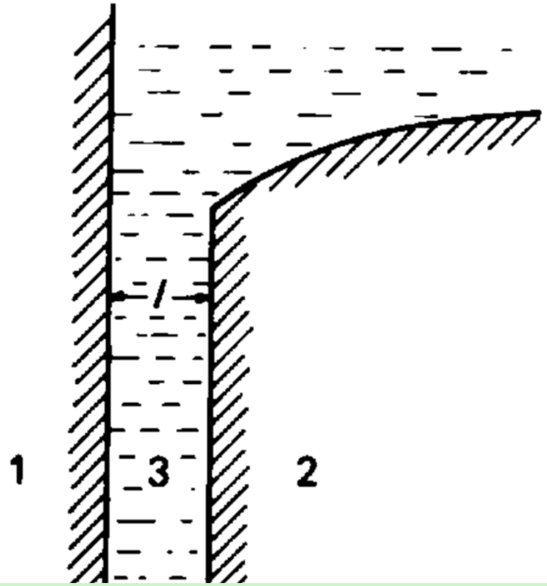
Difference between the Van der Waals force and the Casimir Force

- $\Delta_4 E$ enters when we consider the retardation effect.

- **One more remark**, $\Delta_2 E \propto R^{-6}$ if $R \ll \lambda$, $\Delta_4 E \propto R^{-7}$

- Hence the retardation effect cause the Casimir force **fall more rapidly with distance** than the force acting in the short-range Van der Waals range.

Repulsive Casimir Force



- The force F acting on unit area of each of the two bodies (media 1 and 2) separated by a gap of width l occupied by medium 3 (Adv. Phys. 10, 165–209 (1961))

$$F = \frac{h}{4\pi^3 c^3} I(l, \varepsilon_1, \varepsilon_2, \varepsilon_3) \qquad (8)$$

$$I = \int_0^\infty \int_1^\infty p^2 \varepsilon_3^{\frac{3}{2}} \xi^3 \left[\frac{(1+\varepsilon_1/\varepsilon_3)(1+\varepsilon_2/\varepsilon_3)}{(1-\varepsilon_1/\varepsilon_3)(1-\varepsilon_2/\varepsilon_3)} \exp\left(\frac{2p\xi l\sqrt{\varepsilon_3}}{c} \right) - 1 \right]^{-1} dp d\xi$$

where the dielectric response function (related to the material polarizability) ε_j of medium j is the function of imaginary frequency,

$$j = 1,2,3$$

Repulsive Casimir Force

- After using some math tools, we can further simplify (8) into the following expression.

$$F = \frac{h}{4\pi^3 c^3} I(l, \varepsilon_1, \varepsilon_2, \varepsilon_3) = \frac{h\bar{\omega}}{16\pi^3 l^3} \tag{9}$$

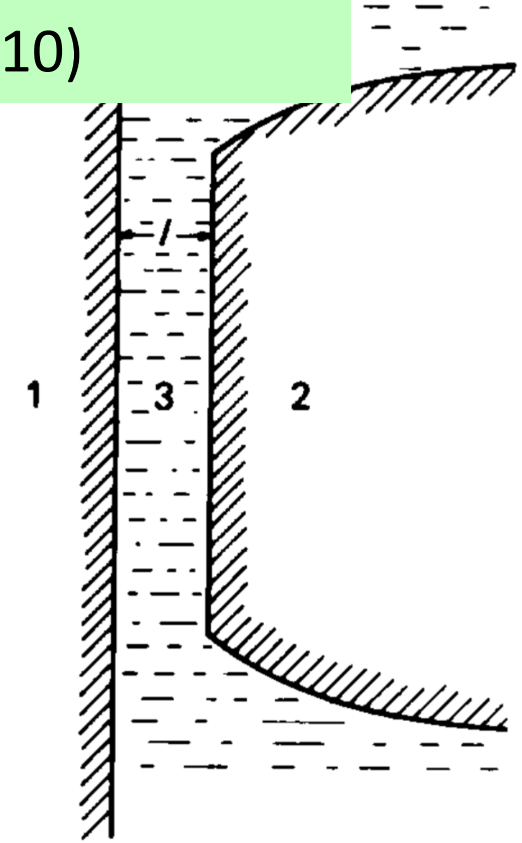
$$\bar{\omega} = \int_0^\infty \frac{\boldsymbol{(\varepsilon_1(i\xi) - \varepsilon_3(i\xi))(\varepsilon_2(i\xi) - \varepsilon_3(i\xi))}}{(\varepsilon_1(i\xi) + \varepsilon_3(i\xi))(\varepsilon_2(i\xi) + \varepsilon_3(i\xi))} d\xi$$

- **What we are interested in is the red part in the integral of $\boldsymbol{\bar{\omega}}$**
- Note that $|\bar{\omega}|$ is some characteristic frequency for the absorption spectra of all three media.

Repulsive Casimir Force

$$\bar{\omega} = \int_0^\infty \frac{\boxed{\boldsymbol{(\varepsilon_1(i\xi) - \varepsilon_3(i\xi))(\varepsilon_2(i\xi) - \varepsilon_3(i\xi))}}}{(\varepsilon_1(i\xi) + \varepsilon_3(i\xi))(\varepsilon_2(i\xi) + \varepsilon_3(i\xi))} d\xi$$

$$\boldsymbol{-(\varepsilon_1 - \varepsilon_3)(\varepsilon_2 - \varepsilon_3)} \qquad (10)$$



- Define the repulsive force to be in positive direction and hence the attractive force is in negative direction.
- When $\varepsilon_1 > \varepsilon_3 > \varepsilon_2$, then (10) is **positive** → **Repulsive force**
- An additional minus sign is put in (10) (Physics doesn't change !)
- Photo adapted from: Adv. Phys. 10, 165–209 (1961)

Main Question:
Can we prove both attractive and repulsive Casimir forces?

Test of Attractive Casimir Effect

Based on Experiment by Mohideen and Roy (1998)

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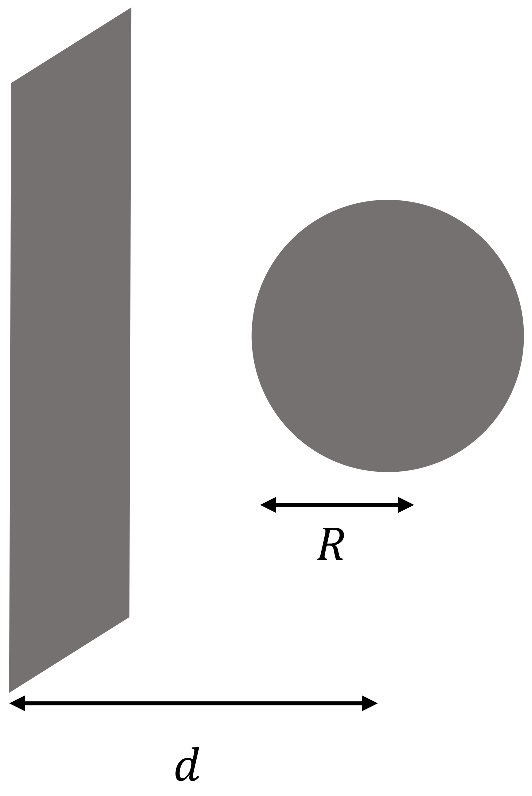
Attractive Casimir: Sphere and Plate Setup

- 2 parallel plates difficult to achieve
- Use sphere and plate

- Casimir effect given by:

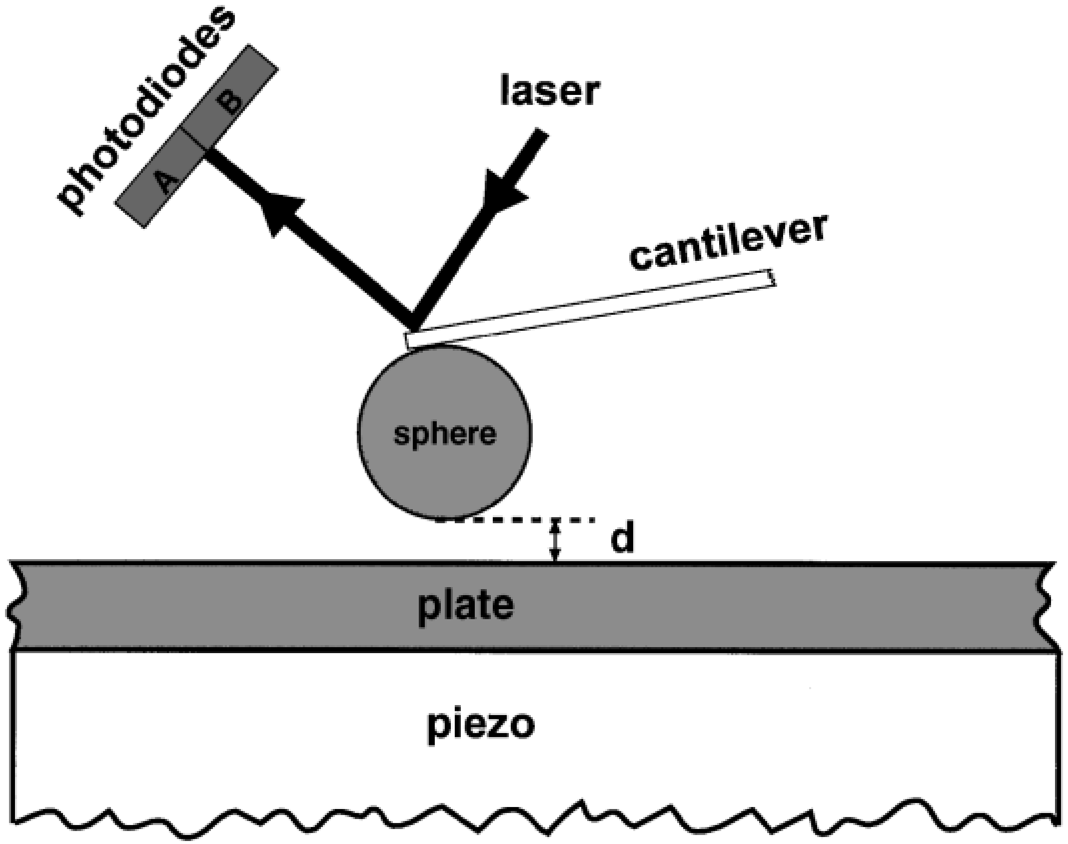
$$\frac{F}{Area} = -\frac{\hbar c \pi^3 R}{360 d^3} \quad (11)$$

- Attractive but inverse cubic



Attractive Casimir: Experimental Setup (AFM)

- Precise force measure: Atomic force microscope (AFM)



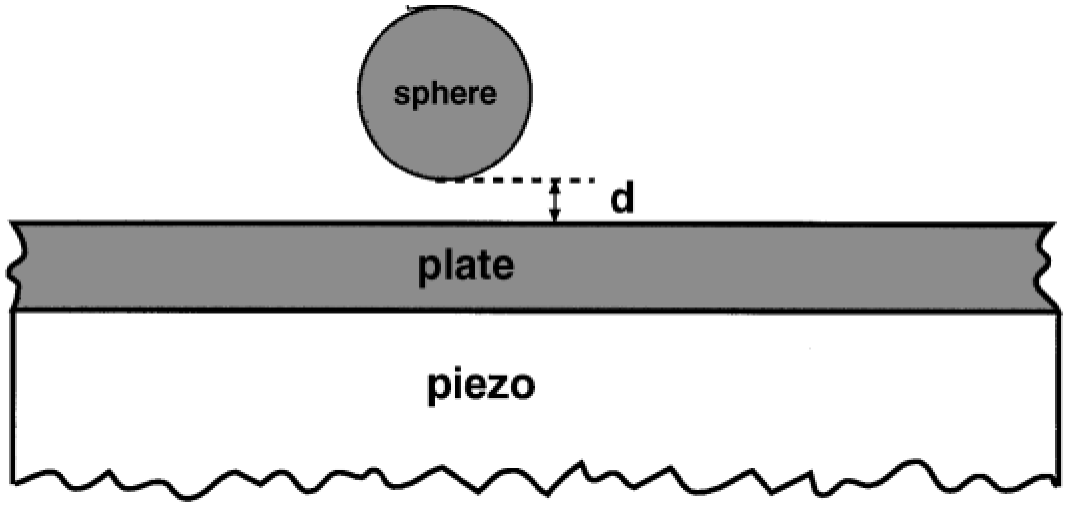
Schematic diagram of the experimental setup. Application of voltage to the piezo results in the movement of the plate towards the sphere.

Adapted from: Mohideen and Roy (1998)

Attractive Casimir: Experimental Setup (AFM)

- Piezo:
- Attached to Al coated plate
- A “lift”
- Apply voltage ⇒ extend

- Control plate-sphere separation



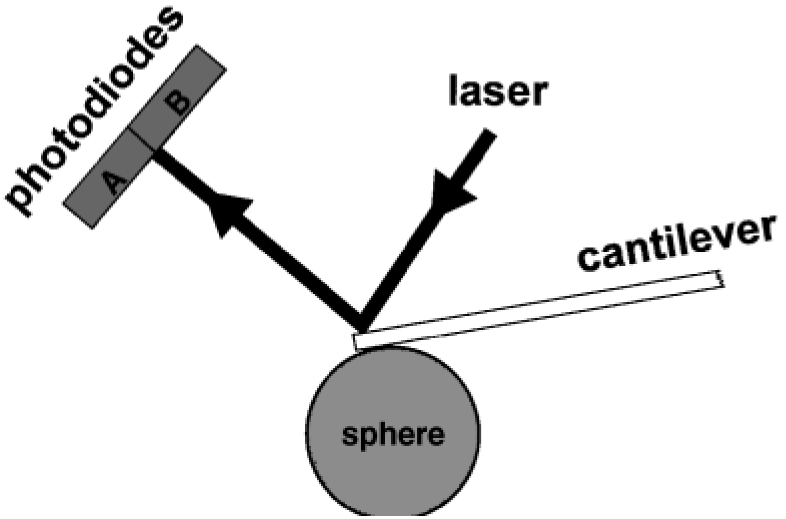
Part of the schematic diagram from slide 19
Adapted from: Mohideen and Roy (1998)

Attractive Casimir: Experimental Setup (AFM)

- Cantilever:
- Attached to Al coated sphere
- Deflect when sphere is attracted

- Laser & photodiodes:
- Shine on cantilever and reflect to photodiodes
- Deflect ⇒ signal difference across photodiodes

- Measure the deflection ⇒ force



Part of the schematic diagram from slide 19
Adapted from: Mohideen and Roy (1998)

Attractive Casimir: Difficulties & Solutions

- Ideal: $F^0 \stackrel{\text{def}}{=} \frac{F}{Area} = -\frac{\hbar c \pi^3 R}{360 d^3}$
- Reality: Finite conductivity, Surface roughness & Finite temperature
- Theoretically take account:
- Finite conductivity: $F^{fc} = F^0 \left[1 - \frac{4c}{d\omega_p} + \frac{72}{5}\left(\frac{c}{d\omega_p}\right)^2\right]$

ω_p: Plasmon frequency of the metal

- Surface roughness: $F^R = F^{fc}\left[1 + 6\left(\frac{A_r}{d}\right)^2\right]$

A_r: Average surface roughness

Attractive Casimir: Difficulties & Solutions

- Finite temperature: $F_c = F^R \left[1 + \frac{720}{\pi^2} f(\xi)\right]$
- $f(\xi) = 1.202\left(\frac{\xi^3}{2\pi}\right) - \left(\frac{\xi^4\pi^2}{45}\right)$
- $\xi = \frac{2\pi k_B T d}{hc} = 0.131 \times 10^{-3}\, d \quad nm^{-1} \qquad (\text{at}\, T = 300K)$

- Casimir << electrostatic:
- Grounded
- Subtract the force due to residual potential

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Attractive Casimir: From Raw Data to Result

- Raw data:
- Photodiodes signal difference S_{pd}
- Piezo extension i.e. distance moved by plate d_{peizo}

- We need:
- Force F
- Distance d

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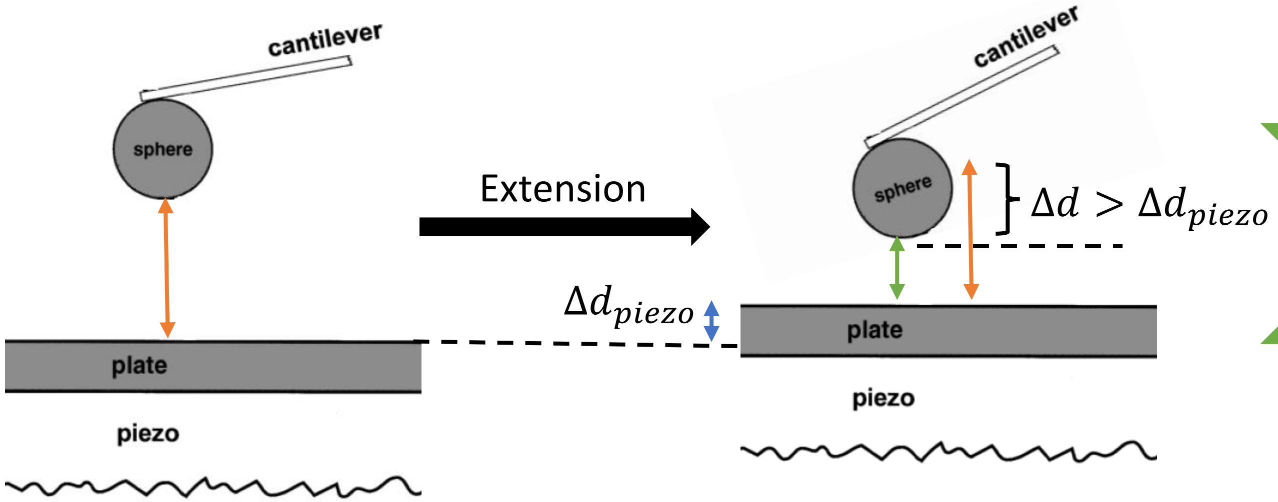
Attractive Casimir: From Raw Data to Result

Calibration

Hooke's const k

Photodiodes signal difference → Cantilever deflection → Casimir force



Piezo extension → Plate-sphere separation

Subtract the deflection

Illustration of how piezo extension differs from plate-sphere separation. Based on schematic diagram from slide 19. Adapted from: Mohideen and Roy (1998)

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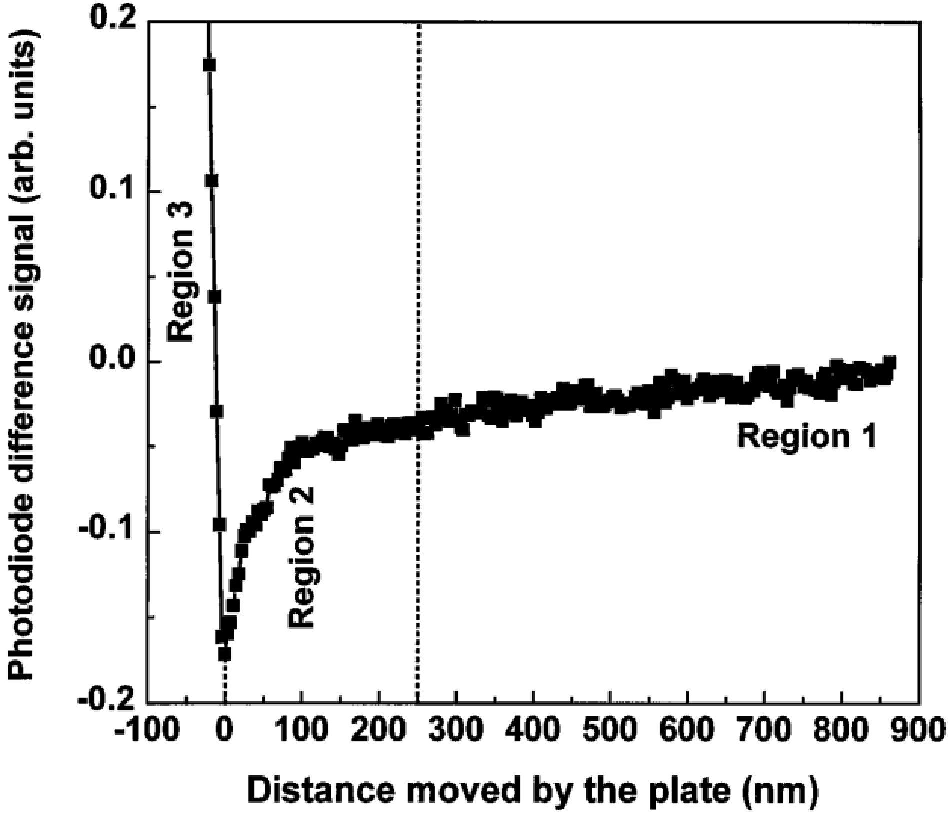
Attractive Casimir: From Raw Data to Result



Signal difference curve (S_{pd}) as a function of distance moved by the plate (d_{piezo}). Adapted from: Mohideen and Roy (1998)

- Contact at $d_{piezo} = 0$
- Region 3: Plate pushing the sphere up & deflect upward

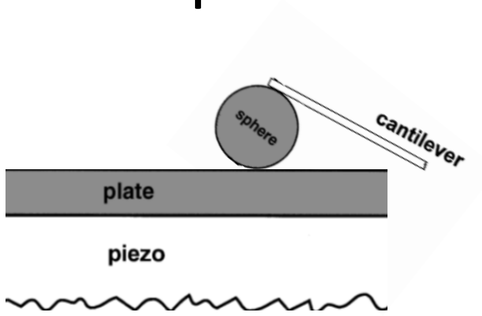
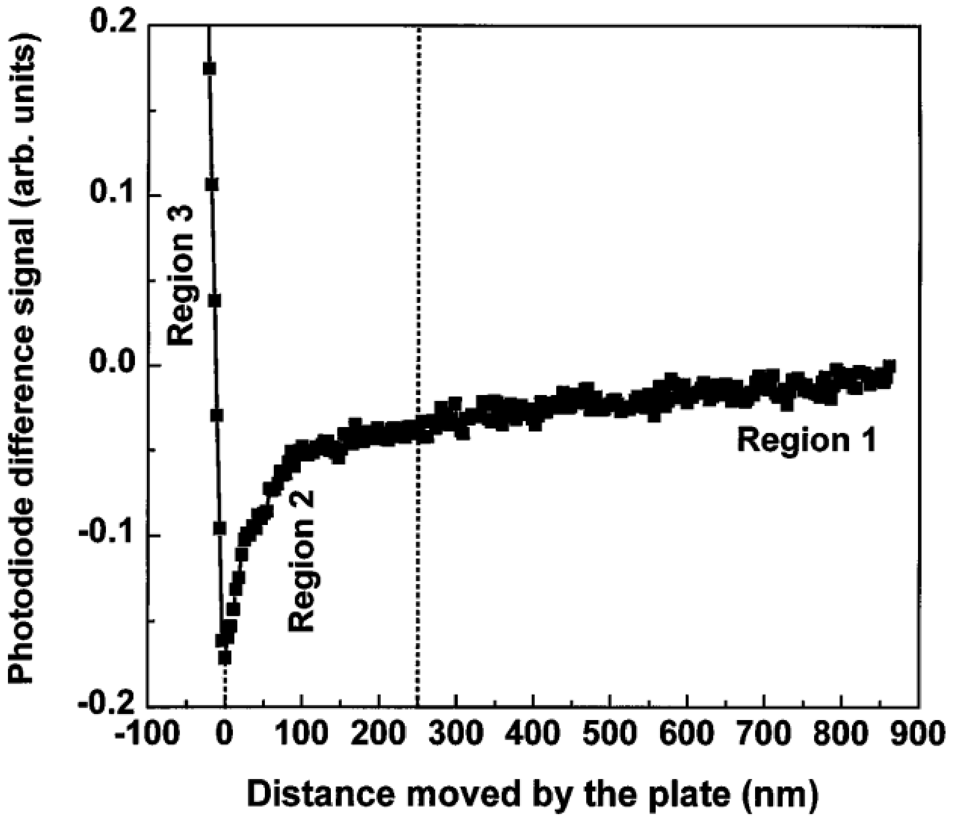


Illustration of region 3. Based on schematic diagram from slide 19. Adapted from: Mohideen and Roy (1998)

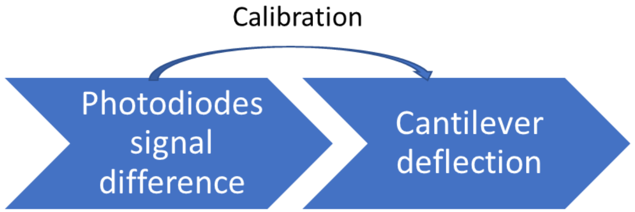
- Region 2: Casimir effect
- Region 1: Scattered light from approaching plate

Attractive Casimir: From Raw Data to Result

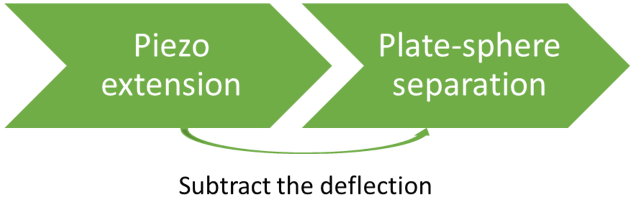


Signal difference curve (S_{pd}) as a function of distance moved by the plate (d_{piezo}).
Adapted from: Mohideen and Roy (1998)

- Region 3: deflection known, = distance moved by plate
- Signal difference to deflection!



- Know deflection
- Piezo extension to plate-sphere separation!



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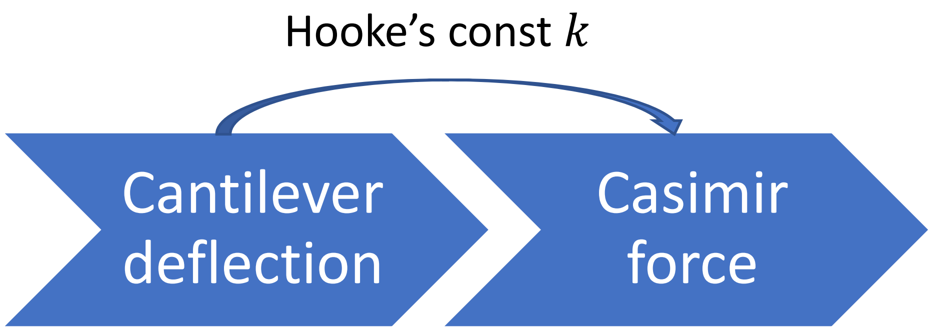
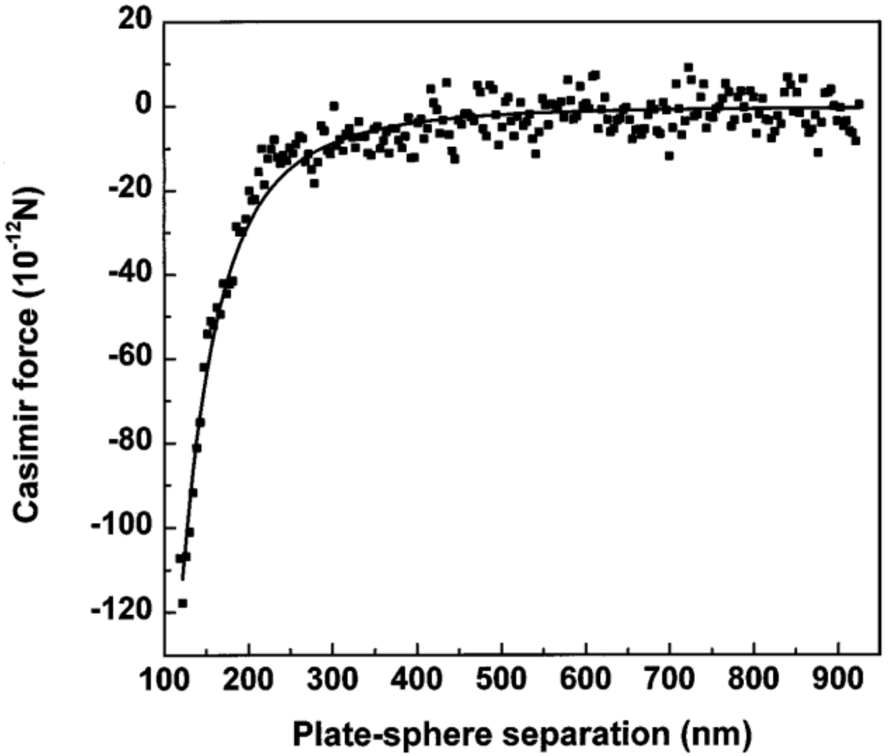
Attractive Casimir: From Raw Data to Result

- Hooke's Constant:
- Use electrostatic force F_{el}
- Apply voltage to sphere and plate
- Measure deflection Δz_{el}

- Known force
- Hooke's constant k! $(F_{el} = k\Delta z_{el})$
- Casimir force! $(F = k\Delta z)$



Attractive Casimir: Result and Discussion

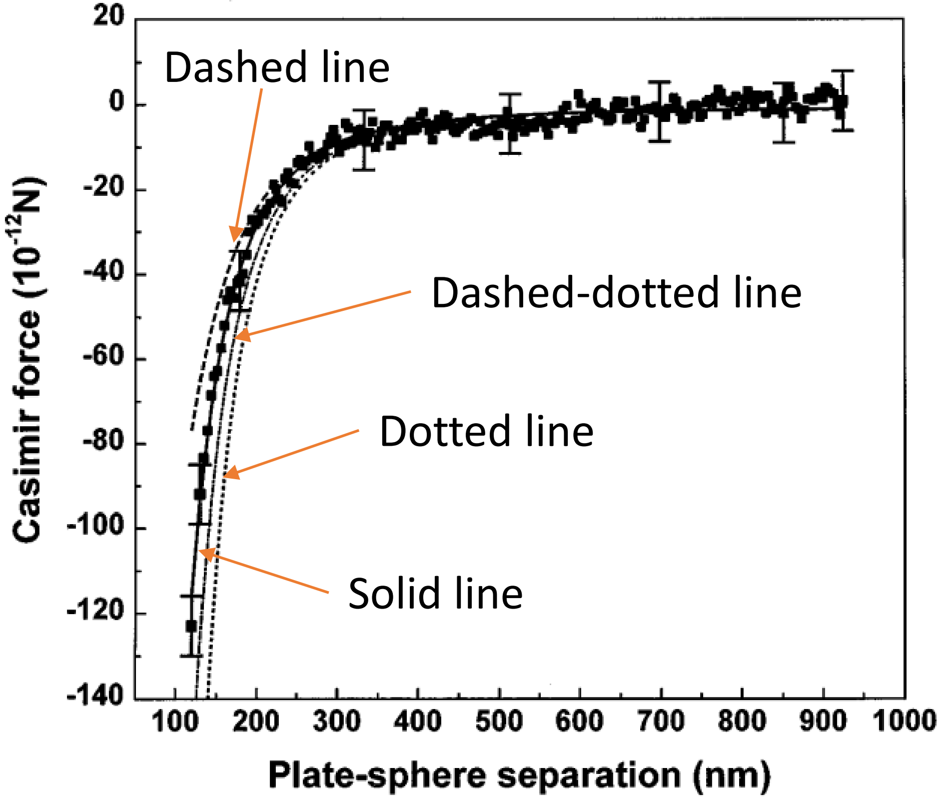


The measured Casimir force as a function of sphere-plate surface separation. Adapted from: Mohideen and Roy (1998)

- Solid line: Casimir effect with finite conductivity, temperature and roughness correction

- Data points follow the corrected Casimir effect well

Attractive Casimir: Result and Discussion



The measured average Casimir force as a function of plate-sphere separation for 26 scans. The error bars show the range of experimental data at representative points.
Adapted from: Mohideen and Roy (1998)

- Repeating for 26 times
- Solid line: Casimir effect with all correction
- Dash-dotted line: without any correction
- Dash line: with conductivity only
- Dotted: with roughness only

- Data points follow the all-corrected Casimir effect the best

Attractive Casimir: Result and Discussion

- Data and all-corrected deviation: 1%
- **Attractive Casimir effect verified**

- Possible improvements:
- Low temperature
- Longer cantilever
- Deflect detection with interfermeter

Test of Repulsive Casimir Force

Based on the experiment performed by Munday, Capasso & Parsegian (Nature, Vol 457, 8 Jan 2009)

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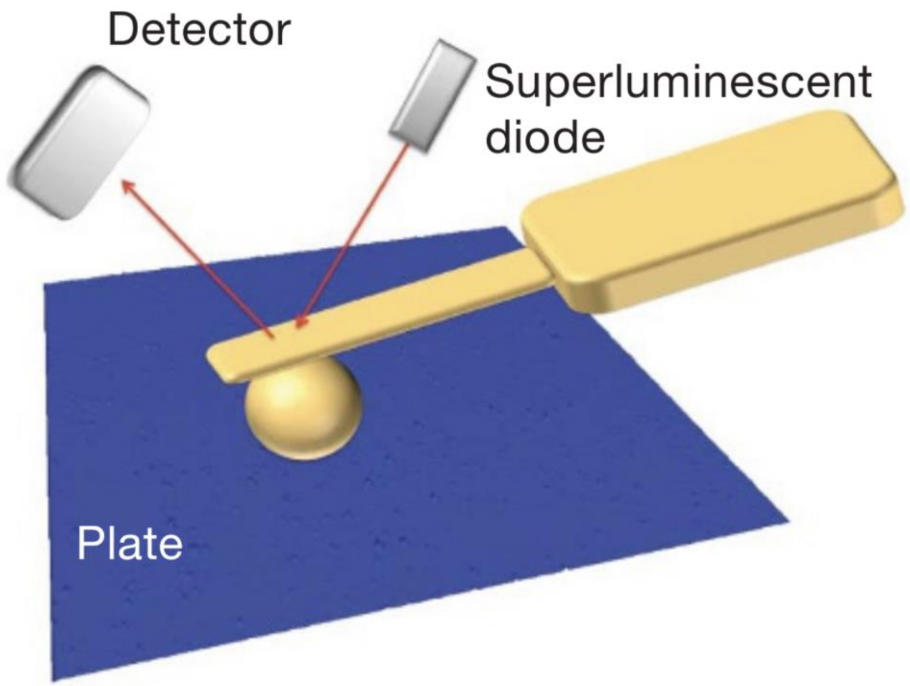
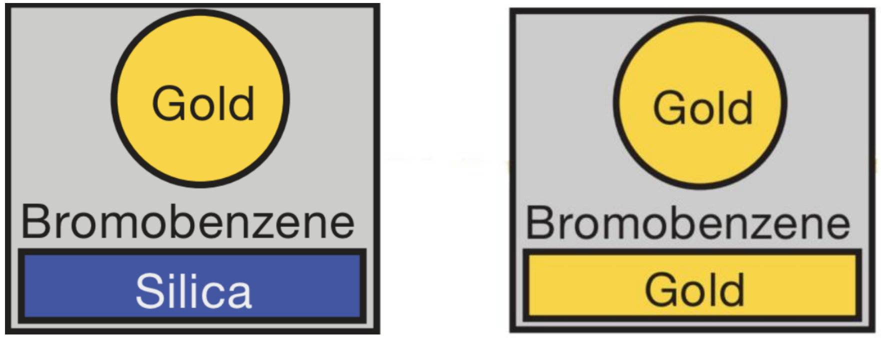
Experimental Methods

- Actually, the setup is **nearly the same**……
- But with a **fluid-filled cell** with **bromobenzene**
- Some details: a 39.8 μm diameter polystyrene sphere coated with a 100 nm think gold film
- Two different setups: one with gold plate, one with silica plate
- Photo adapted from Munday, Capasso & Parsegian (Nature, Vol 457, 8 Jan 2009)



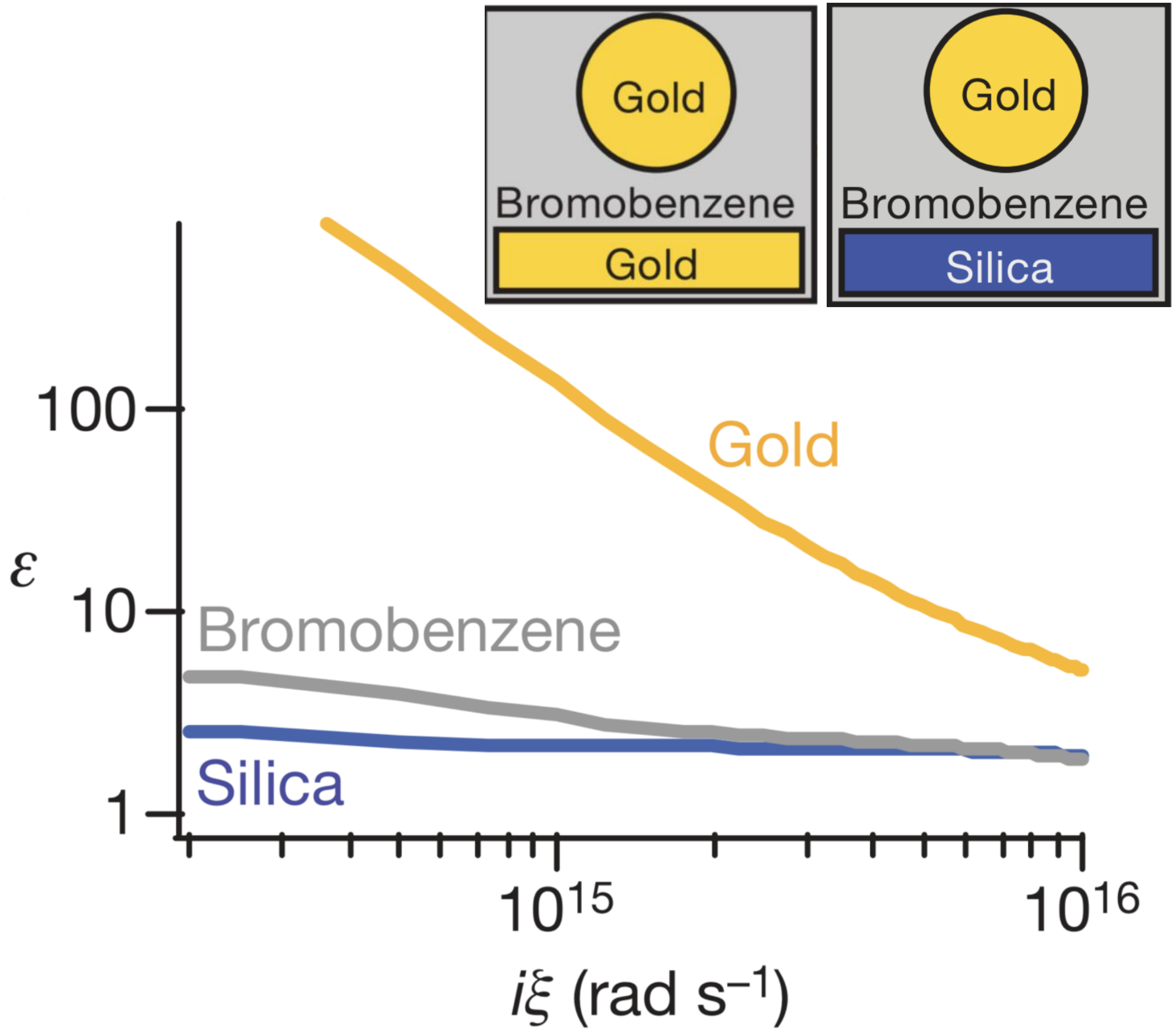


Experimental Methods

- The graph is plotted as dielectric response function ε against the imaginary frequency $i\xi$
- $\varepsilon_{gold} > \varepsilon_{bromobenzene} > \varepsilon_{silica}$

Recall:

- When $\varepsilon_1 > \varepsilon_3 > \varepsilon_2$, then (10) is **positive** → **Repulsive force**
- If $\varepsilon_1 = \varepsilon_2$ → **Attractive force**
- Plot adapted from Munday, Capasso & Parsegian (Nature, Vol 457, 8 Jan 2009)



$$-(\varepsilon_1 - \varepsilon_3)(\varepsilon_2 - \varepsilon_3) \qquad (10)$$

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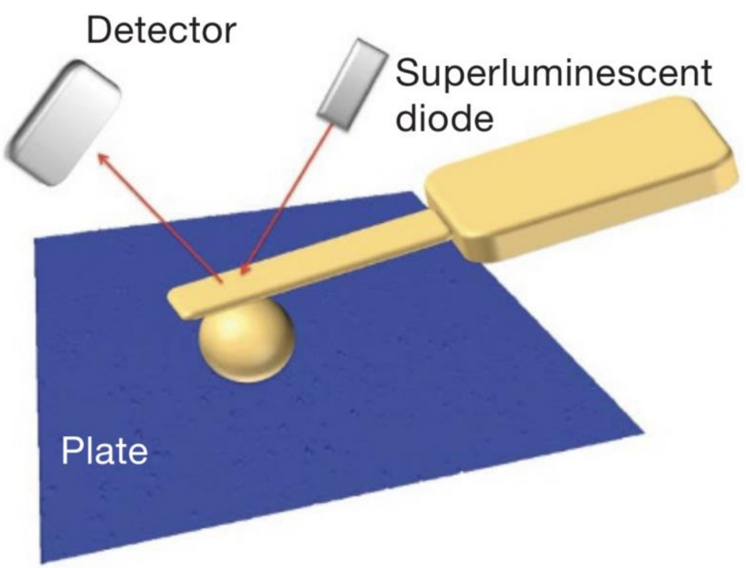
Experimental Methods

- Light from a diode is reflected off the back of the cantilever and is used to **monitor the cantilever's bending**.
- A change in the detector signal that monitors the difference in light intensity between the top-half and the bottom half of the detector.
- The difference signal is proportional to the force
- Photo adapted from Munday, Capasso & Parsegian (Nature, Vol 457, 8 Jan 2009)

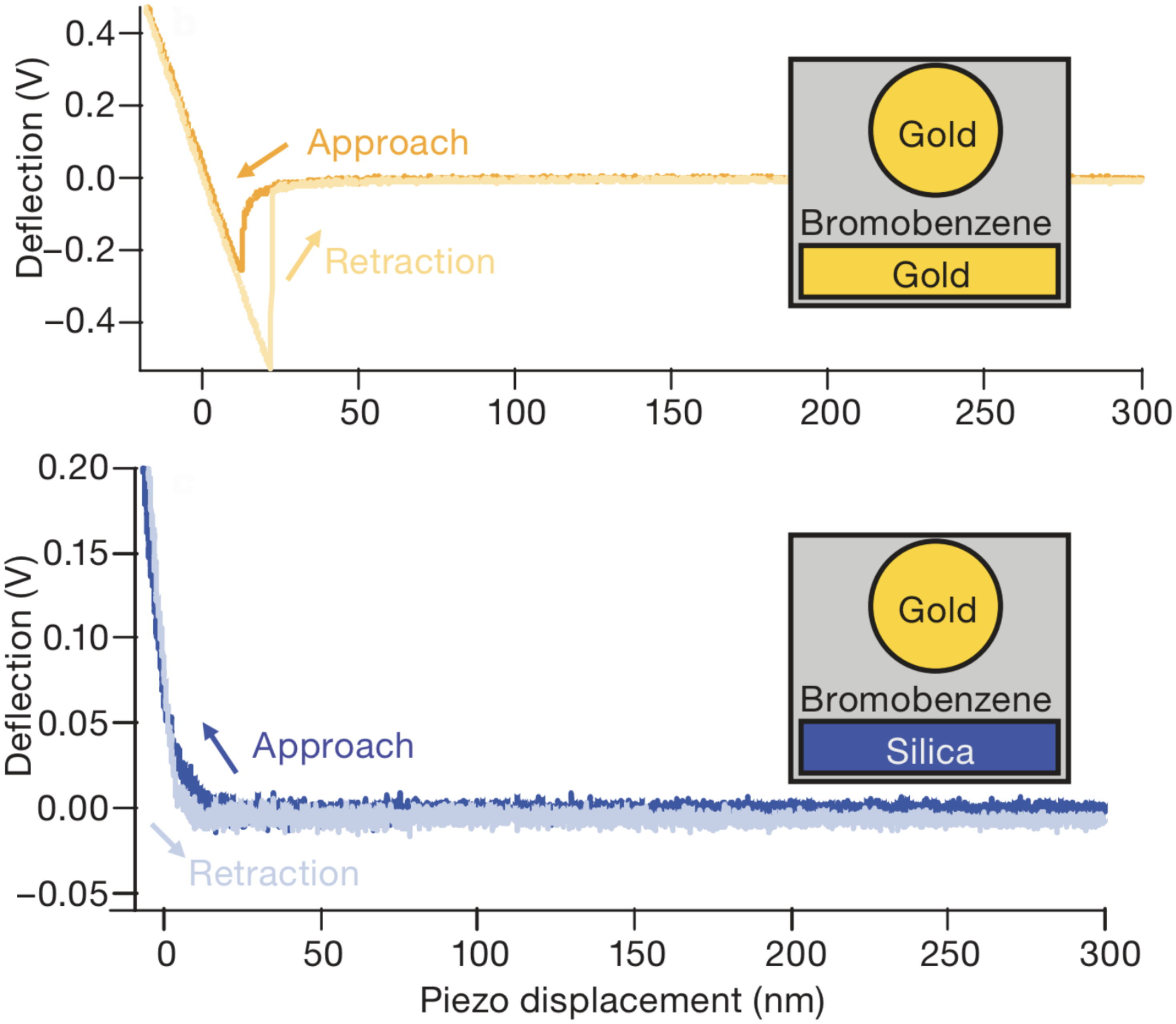


Experimental Methods

- Cleaning procedures are performed on all surfaces before the experiment.
- Electrostatic force microscopy is performed on the samples to ensure the **surface charge effects** are small and will not mask the Casimir force (experimental difficulty)
- No evidence of excess charge accumulation is found on the plate
- The whole setup is assembled and allowed to equilibrate for 1 hr before the measurements
- Measured at room temperature

Data Analysis

- Piezo speed: $45\ \mathrm{nm\ s^{-1}}$
- Approach and retraction
- Note negative deflection $\rightarrow$ attractive force
- Photo adapted from Munday, Capasso & Parsegian (Nature, Vol 457, 8 Jan 2009)



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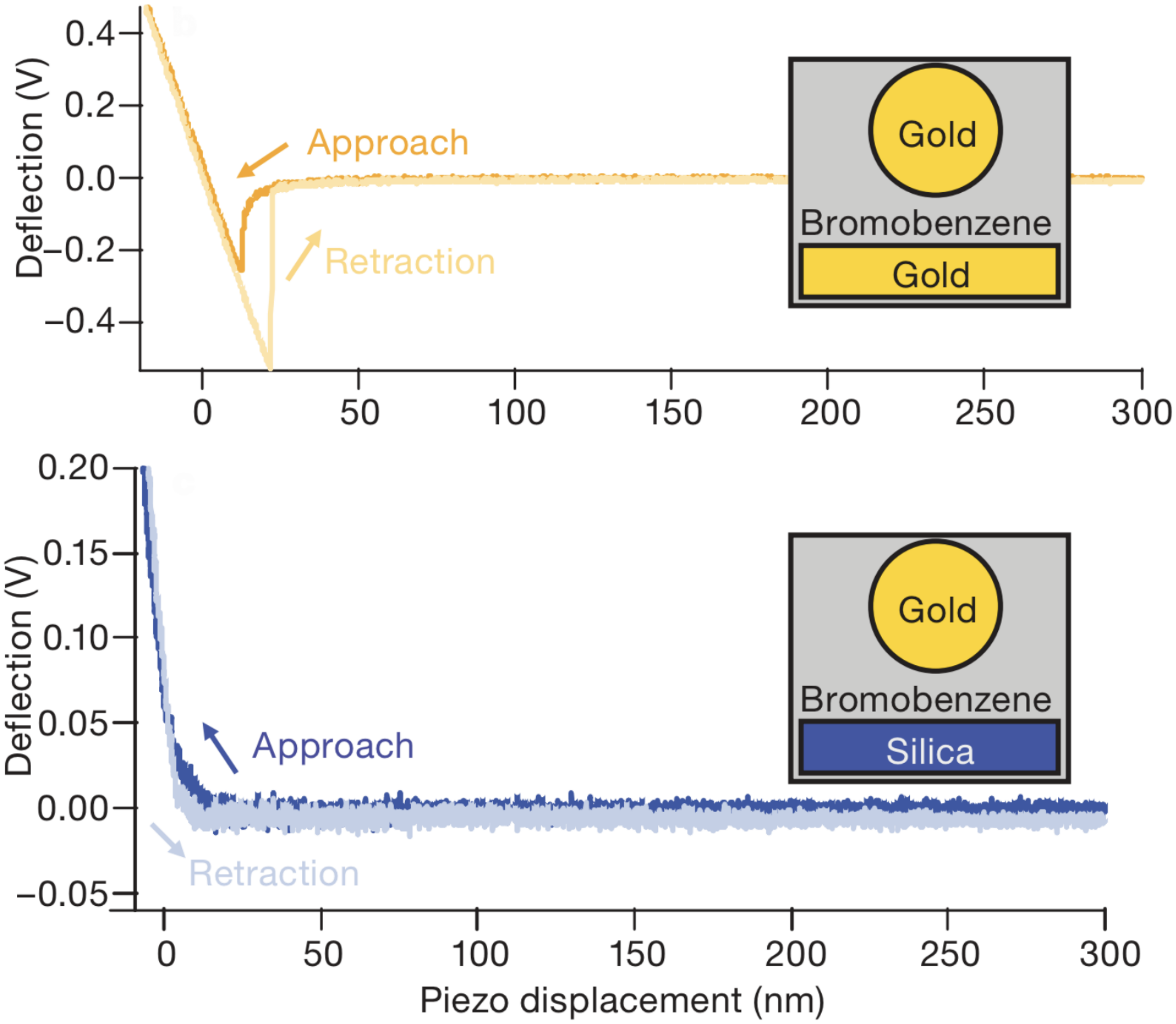
Data Analysis

- CANNOT be the result of hydrodynamic force
- CANNOT be due to charge trapped on silica (Image charge on metal sphere $\rightarrow$ attraction)
- Photo adapted from Munday, Capasso & Parsegian (Nature, Vol 457, 8 Jan 2009)



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Data Analysis

- The detector signal is converted to a force signal by calibration with the hydrodynamic force
- By Munday (Rev. A 78, 032109 (2008)), the total force F_{total} (for $R \gg d$)

$$F_{total}(d,v) = F(d) + F_{hydro}(d,v) + Ad + B \qquad (12)$$

$$F_{hydro}(d,v) = \frac{6\pi\eta v}{d} R^2 \propto v \qquad (13)$$

where F_{hydro} is the hydrodynamic force, F is the Casimir force, A and B are constant, d is the separation of a sphere of radius R and a plate, η is the fluid viscosity and v is the velocity of the plate relative to the sphere.

Data Analysis

$$F_{total}(d,v) = \cancel{F(d)} + F_{hydro}(d,v) + \cancel{Ad} + \cancel{B} \quad (12)$$

$$F_{hydro}(d,v) = \frac{6\pi\eta v}{d} R^2 \propto v \quad (13)$$

From (12), (13), we can get that

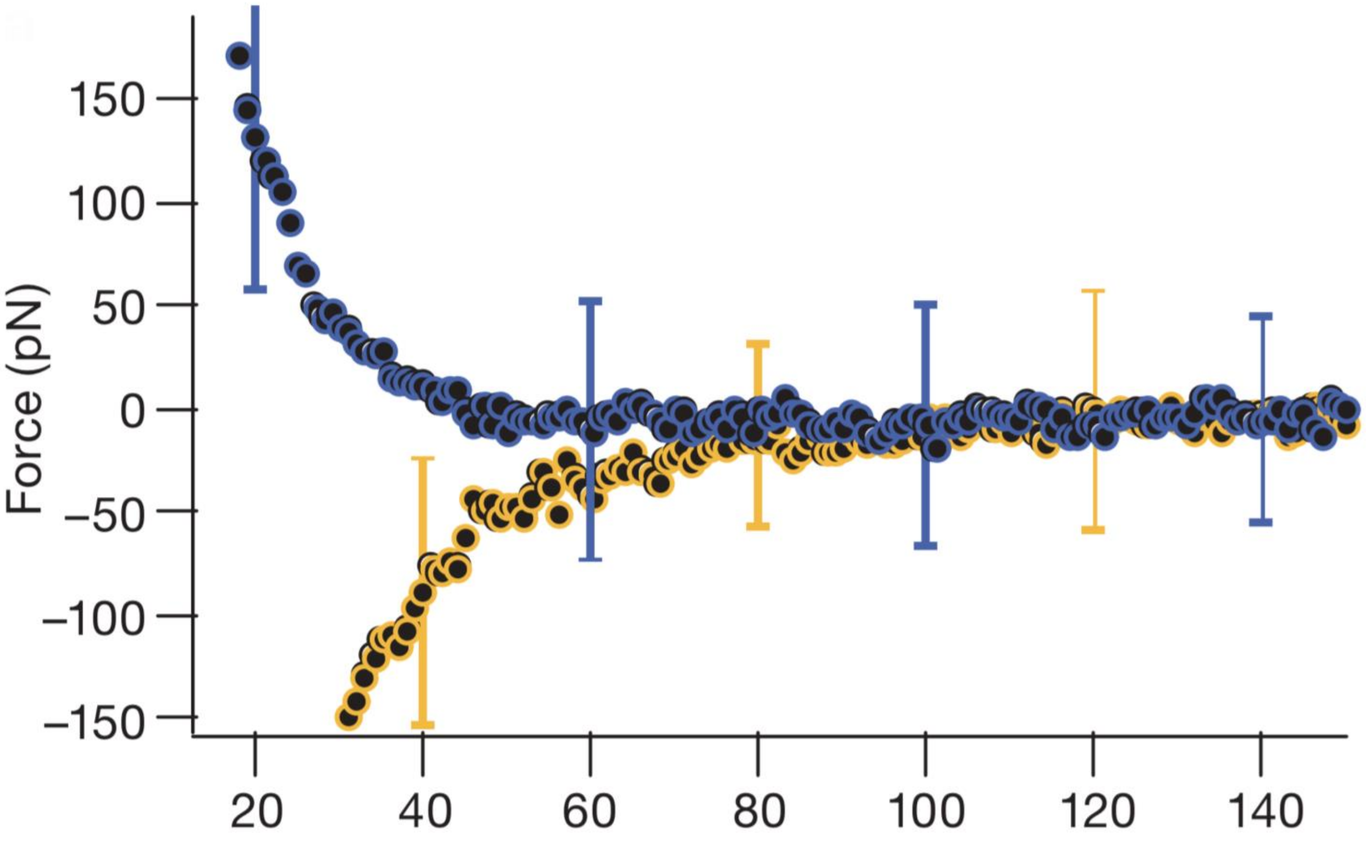
$$F_{hydro}(d,v) = F_{total}(d,v_1) - F_{total}(d,v_2) \quad (14)$$

where $v = v_1 - v_2$

- By performing measurements at corresponding piezo velocity v_1, v_2, we can get the hydrodynamic force at piezo velocity v.
- We can determine $Ad + B$ by measuring the force signal at large distance d

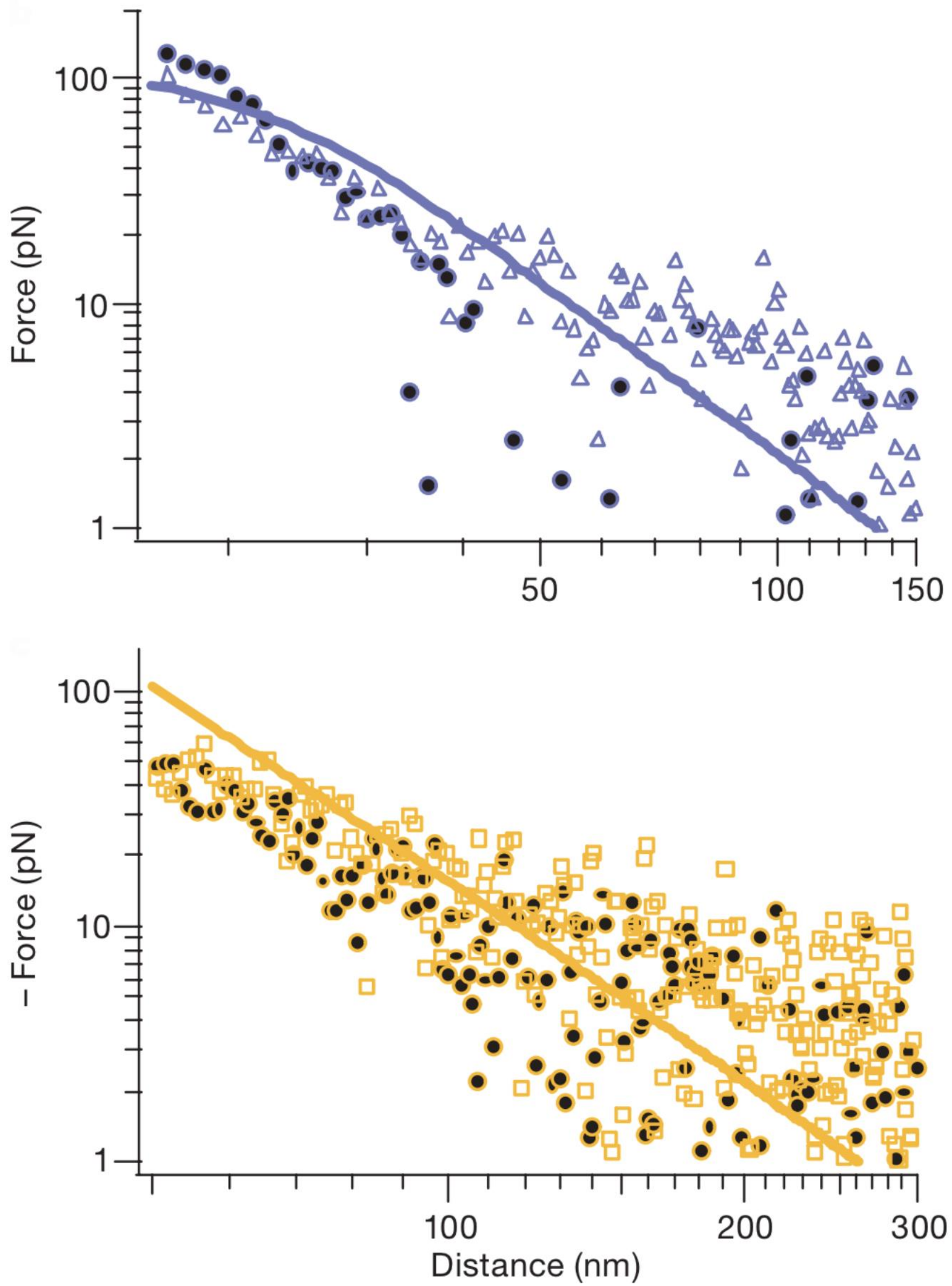
Data Analysis

- Note that the x-axis of the graph is distance in unit of nm.
- The blue (orange) circles correspond to the average force from 50 runs between the gold sphere and the silica (gold) plate
- Positive: repulsive
- Attractive force > repulsive force
- Plot adapted from Munday, Capasso & Parsegian (Nature, Vol 457, 8 Jan 2009)



Data Analysis

- Whole set experiment is repeated using another sphere with the same diameter and another sets of gold and silica plates.
- Blue: silica plate ; orange: gold plate
- The solid lines: theoretical case with surface roughness correctness
- The two plots are in log-log scale
- Plot adapted from Munday, Capasso & Parsegian (Nature, Vol 457, 8 Jan 2009)



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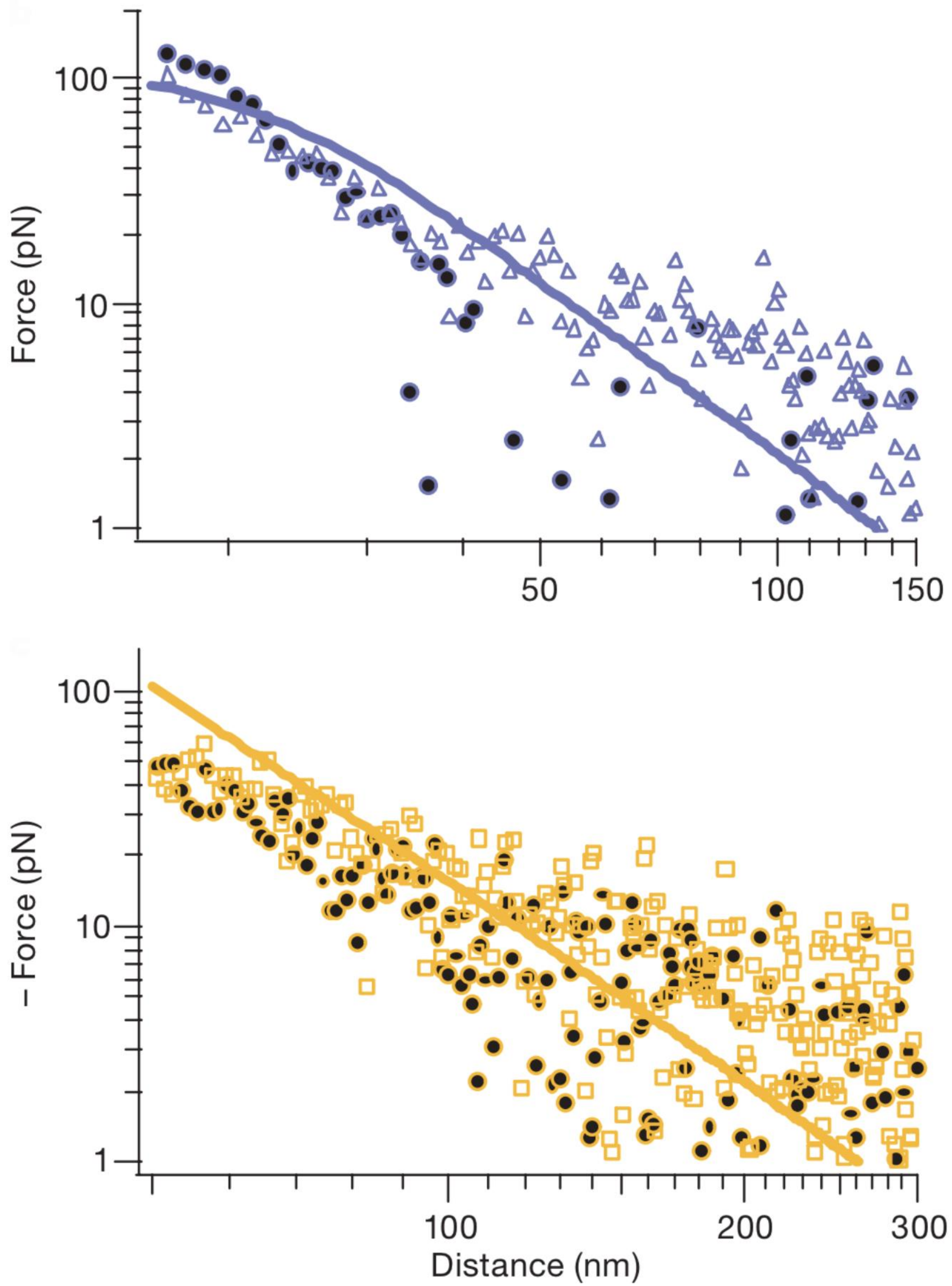
Data Analysis

- Discrepancy mainly due to the uncertainty in optical properties
  - ➢ Measurements of the optical properties of bromobenzene for a large spectral range isn't available.
  - ➢ Optical properties are modified for very thin films
- Surface roughness correction fail at small separation
- Force below 10 pN
- Plot adapted from Munday, Capasso & Parsegian (Nature, Vol 457, 8 Jan 2009)



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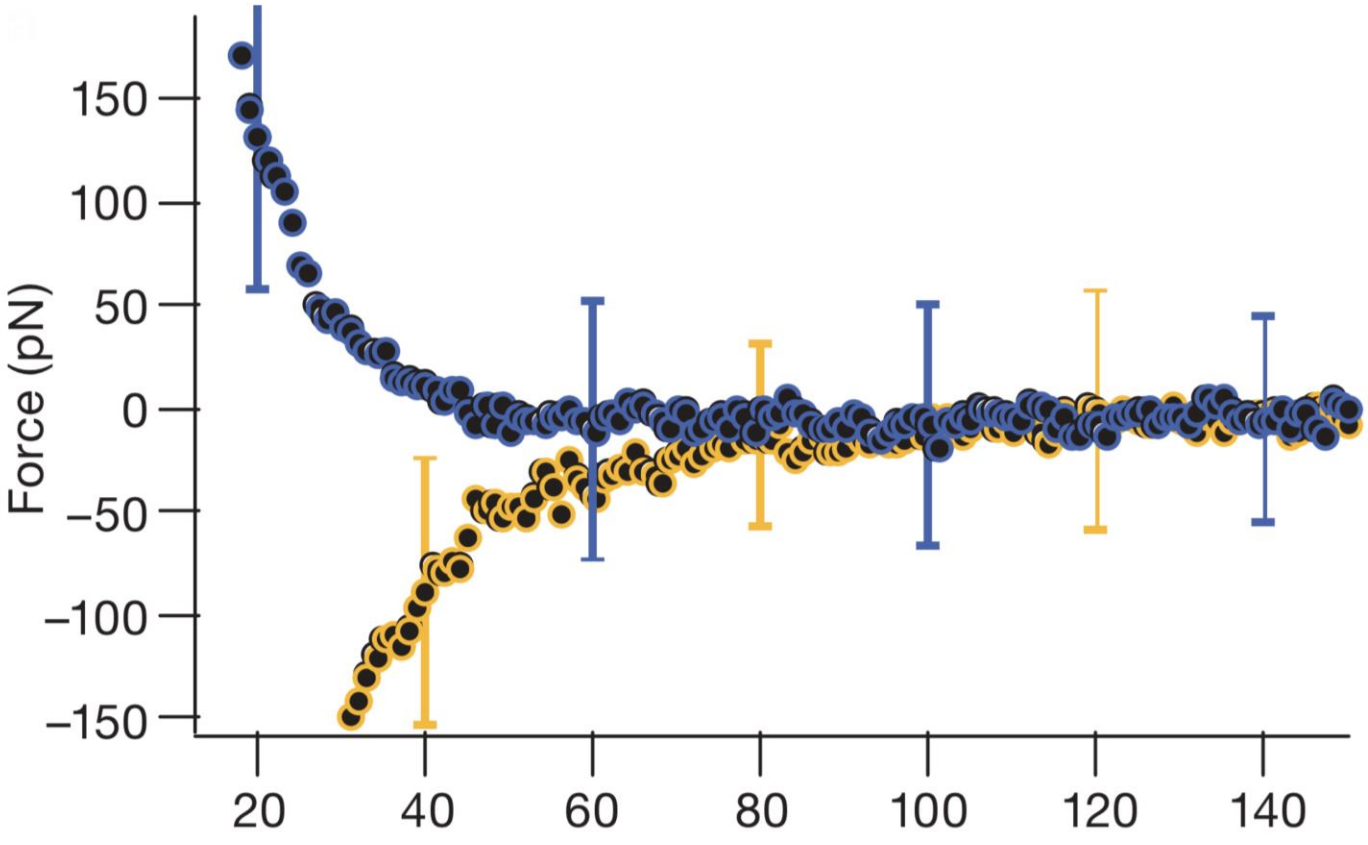
Data Analysis

- Repulsive effect is still verified!

- Plot adapted from Munday, Capasso & Parsegian (Nature, Vol 457, 8 Jan 2009)



Application & Importance of Casimir Effect:

- Separation measurement: $$\frac{F}{Area} = -\frac{\hbar c \pi^2}{240 d^4} \quad (7)$$
- Repulsive Casimir effect:
  - Ultra low friction device
  - Quantum levitation
- Crucial in micro/nano devices e.g. integrated circuit
  - 10 nm separation $\Rightarrow$ 1 atm pressure
- Maybe helpful in theoretical topics?
  - Dark energy
  - Topology of universe

Conclusion

- Theory behind
- Two experiments
- Application

Reference

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Thanks

Questions?